XIXth INTERNATIONAL CONFERENCE ON PUBLIC EDUCATION
GENEVA 1956

TEACHING OF MATHEMATICS

IN SECONDARY SCHOOLS

COMPARATIVE STUDY

AFGHANISTAN, AUSTRALIA, BELGIUM, BULGARIA, BYELORUSSIA, CAMBODIA, CANADA, CEYLON, CHILE, CHINA (REPUBLIC OF), COLOMBIA, COSTA RICA, CUBA, CZECHOSLOVAKIA, DENMARK, DOMINICAN REPUBLIC, ECUADOR, EGYPT, FINLAND, FRANCE, GERMAN FEDERAL REPUBLIC, GREECE, GUATEMALA, HAITI, HONDURAS, HUNGARY, INDIA, IRAQ, IRELAND, ISRAEL, ITALY, JAPAN, LAOS, LIBERIA, LUXEMBURG, MONACO, NEPAL, NETHERLANDS, NEW ZEALAND, NICARAGUA, NORWAY, PAKISTAN, PANAMA, PERU, PHILIPPINES, POLAND, PORTUGAL, ROUMANIA, SALVADOR, SPAIN, SWEDEN, SWITZERLAND, SYRIA, THAILAND, UKRAINE, UNION OF BURMA, UNION OF SOUTH AFRICA, UNION OF SOVIET SOCIALIST REPUBLICS, UNITED KINGDOM, UNITED STATES, VIETNAM, YUGOSLAVIA

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INTRODUCTION

One topic discussed by the XIIIth International Conference on Public Education convened jointly in 1950 by Unesco and the International Bureau of Education, was the introduction to mathematics in primary schools. As working document for this item on its agenda, the Conference had at its disposal a volume [1] based on the information received by the Bureau from the Ministries of Education of forty-six countries.

The present volume on the teaching of mathematics in secondary schools is a natural sequence to that of 1950, and is based on a comparative study of the replies from sixty-two countries. It should be noted that the inquiry was limited to general secondary schools, and excluded any consideration of the conditions of mathematics teaching in secondary institutions providing vocational training of a special charakter, where the teaching of mathematics, like that of other subjects in theses chools, takes into account the requirements of the particular vocation.

It is hoped that the information presented in the present volume will assist the XIXth International Conference on Public Education to formulate a recommendation which will serve to improve the teaching of a subject of increasing importance.

Grateful acknowledgment is herewith made to the Ministries of Education of the sixty-two countries replying to the questionnaire of the International Bureau of Education and supplying the documents which have made this study possible.

Madeleine VELUZ-PAGANO

Research Division
International Bureau of Education

[1] " Introduction to Mathematics in Primary Schools ". Paris, Unesco ; Geneva, International Bureau of Education, Publication No. 121, 1950.

QUESTIONNAIRE ADDRESSED TO THE MINISTRIES OF EDUCATION

I. Place of Mathematics in Curriculum

1. (a) In what years of the various sections, divisions or stages of secondary schools are mathematics taught?
   (b) To what ages do these years correspond?
   (c) How many hours a week are devoted to mathematics in each class and what is the proportion of the time given to this subject in relation to the rest of the time-table?
2. Under what subject headings are mathematics taught in each class?
3. Is mathematics teaching sometimes optional? If so, kindly indicate in which cases.
4. Is special importance accorded to the teaching of mathematics in comparison with other subjects:
   (a) in promotion examinations;
   (b) in the secondary school leaving examinations?

II. Aims of Mathematics Teaching

5. What are the aims assigned to the teaching of mathematics in the official instructions regulating the different sections of secondary education?

III. Syllabuses

6. What are, very briefly stated, the principal questions treated in the mathematics syllabus for each class in the different sections of secondary schools?
7. Do the syllabuses include the history of mathematics and an introduction to axioms?

IV. Methods of Teaching Mathematics

8. (a) Are there any official instructions or suggestions concerning the methods to be used in teaching mathematics in secondary schools?
   (b) If so, kindly give a brief summary of them.
   (c) If not, what methods are most commonly used?
9. Are there any special instructions bearing on teaching methods for mathematics at the transfer stage from primary to secondary school?

10. How is coordination established between the teaching of mathematics and other subjects taught in secondary schools (physics, economics, etc.)?

11. Kindly mention the teaching materials and auxiliary aids (films, lantern slides, etc.) that may possibly be used in mathematics teaching.

12. (a) Does the mathematics syllabus include applied mathematics, and of what nature?
    (b) To what extent are activity methods used in connection with the practical exercises (handicrafts, surveying, etc.) preceding formal teaching?

13. Are there official instructions concerning the choice and use of textbooks for the teaching of mathematics in secondary schools? If so, what are their requirements? In case where textbooks are not used, by what are they represented?

V. Teachers

14. What special training (scientific, pedagogical and practical) is required of teachers of mathematics in secondary schools?
    What degrees and diplomas are required?

15. Is there difficulty in recruiting mathematics teachers? If so, what is the reason for this and what measures have been taken to overcome the difficulty?

16. What means are employed to keep serving teachers of mathematics up-to-date in their knowledge (interchange of teachers, lectures and special courses, seminars and study courses, associations of mathematics teachers, educational journals, travel scholarships, etc.)?

VI. Miscellaneous

17. (a) Are any modifications contemplated in the teaching of mathematics in secondary schools? If so, kindly indicate their nature.
    (b) To what extent does the evolution of modern mathematics affect secondary education?

18. Are any experiments or studies in the teaching of mathematics being carried out? If so, kindly indicate their nature.

19. What are the most recent books published in your country on the methods of teaching mathematics in secondary schools?

20. Kindly add any supplementary information deemed useful.

21. If possible, kindly add to your reply:
    (a) the mathematics syllabuses at present in force;
    (b) a selection of the textbooks most frequently used in your country for the teaching of mathematics in secondary schools;
    (c) any other documents of particular interest to this subject.

All documents sent will be placed in the Permanent Exhibition of Public Education and will form part of the International Education Library in Geneva.

GENERAL SURVEY

I. PLACE OF MATHEMATICS IN CURRICULUM

Designation and Required or Non-Required Status of Subject

Mathematics forms a regular part of the curriculum of all the general secondary schools of the sixty-two countries participating in the present inquiry.

In regard to the designation of the subject, the countries participating fall into two more or less equal groups: (a) those countries where the subject is broadly designated "mathematics" or "mathematics and geometry", in all the years of secondary education; (b) those countries where the subject is designated in each year according to the branch or branches of the subject taught in that year; and it is this group for which some explanation is necessary.

With regard to this second group, it is of interest to note that the subject is commonly designated "arithmetic" in the first years of secondary education, and then, in the subsequent years, "mathematics" or "algebra". This change of designation gives some indication of the transition, frequently referred to, especially in connection with methodology, in almost all the replies, from emphasis on the concrete to emphasis on the more abstract.

The practical bias of the teaching given at lower secondary level is indicated in some of the countries of the second group by the use of qualifying adjectives. The term *aritmética práctica*, for example, is used in Honduras, *aritmética razonada y comercial* in Nicaragua, and "social mathematics" in Norway.

The terms in common use at the upper secondary level in the countries of the second group include "infinitesimal calculus", "cosmography", "mathematical analysis", "elements of higher mathematics", and "mechanics". In some cases the different aspects of geometry are each given their own name, and these names then show a certain progression: "plane geometry", "solid geometry", "trigonometry", "analytical geometry".

In regard to mechanics, it is noteworthy that this branch is specifically named in relatively few of the countries. The cases, however, in which it forms part of the mathematics syllabuses are more numerous, and in all the countries it must certainly form part of the physics courses. It would evidently be arbitrary to place mechanics in a watertight compartment, since it is not only associated and correlated with, but forms an integral part of, both mathematics and physics.

It is thus clear that in the countries of the second group the varying designation of mathematics in the different years of secondary education indicates the gradually advancing nature of the teaching, and the existence of branches with mathematical aspects which must be grasped for a mastery of the whole field.

In all the countries mathematics is to the forefront in secondary education. Its importance is revealed by an analysis of its status as a required or non-required subject, above all in the science sections.

The details of that status are given in the table below. Analysis of that table shows the following :

(a) In two thirds of the countries mathematics is a required subject in all years and in all sections and classes of general secondary education.

(b) In all the countries except two (Philippines and Vietnam, where mathematical instruction begins only in the 2nd year) mathematics is a required subject throughout the lower secondary cycle.

(c) A required subject in the lower secondary cycle, mathematics in some ten of the countries becomes an optional subject in the upper cycle, or is dropped altogether in the last year or years of certain sections.

(d) In some twelve of the countries mathematics remains a required subject in the upper secondary cycle only for those pupils taking their school leaving certificate in science.

(e) Differentiation on the basis of sex, in respect of whether mathematics is a required or non-required subject, is relatively rare. In Denmark, in the *realklasse*, all the mathematical branches taken are optional for girls, but for the boys this is not the case, while in India arithmetic, algebra and geometry are required for boys, but only the first of these three branches for girls.

(f) Optional, additional courses in mathematics are offered in the upper secondary cycle in a few of the countries (France, Laos, Monaco, Nepal, Pakistan). Pupils with aptitude and an interest in science are thus enabled to do work at a higher level than is required by their curriculum. In the U.S.S.R. and other countries in Eastern Europe, moreover, such pupils can optionally pursue their mathematical studies outside class hours, in the school mathematics societies.

AFGHANISTAN. Arithmetic, algebra, geometry ; also (in the upper years) trigonometry, descriptive geometry, mechanics. Compulsory throughout (six-year course, pupils 11 to 18).

AUSTRALIA. Arithmetic, algebra, geometry (1st and 2nd years) ; arithmetic, algebra, geometry, trigonometry (3rd year) ; algebra, geometry, trigonometry, differential calculus, plane analytic and algebraic geometry, in some cases mechanics (4th and later years). Compulsory in the first two or three years, optional in the 4th and later years (four-year course in Queensland, six-year in Victoria, five-year in remaining states ; pupils 12-13 to 17-18).

BELGIUM. *Mathématiques*. Compulsory throughout (six-year course, pupils 12 to 18).

BULGARIA. Algebra, plane geometry (1st and 2nd years) ; algebra, stereometry, trigonometry (3rd year) ; algebra, trigonometry (4th year). Compulsory throughout (four-year course, pupils 14 to 17).

BYELORUSSIA. Arithmetic (1st year) ; arithmetic, algebra, geometry (2nd year) ; algebra, geometry (3rd and 4th years) ; algebra, geometry, trigonometry (5th and 6th years). Compulsory throughout (six-year course, pupils 12 to 17).

CAMBODIA. Mathematics. Compulsory throughout (seven-year course, pupils 14 to 21).

CANADA. Mathematics. Secondary education generally covers a period of six or seven years, the age range of the pupils being 13 to 18. In most of the provinces mathematics is a required subject in the first three years, and thereafter optional except for pupils preparing for university studies.

CEYLON. Arithmetic, geometry, algebra; also (in the last four years for pupils specializing in mathematics) advanced and applied mathematics. Compulsory throughout (seven-year course, pupils begin at 11).

CHILE. *Matemáticas*. Compulsory throughout (six-year course, pupils 11 to 18).

COLOMBIA. *Aritmética* (1st and 2nd years); *álgebra y contabilidad* (3rd year); *álgebra y geometría plana* (4th year); *geometría del espacio y trigonometría* (5th year). Compulsory during first five years (six-year course, pupils 13 to 17).

COSTA RICA. *Matemáticas*. Compulsory throughout (five-year course, pupils 13 to 17).

CUBA. *Matemáticas* (first four years); *ampliacón de matemáticas* (5th year, science section). Compulsory in first four years, and in the 5th year for the science section (five-year course, pupils 12 to 17).

CZECHOSLOVAKIA. Mathematics. Compulsory throughout (six-year course, pupils 11 to 17).

DENMARK. Algebra, geometry, arithmetic (lower cycle and the *realklasse*); mathematics (upper cycle). Compulsory throughout lower cycle (four-year course, pupils 11 or 12 to 15 or 16); algebra and arithmetic compulsory for boys and optional for girls, geometry optional for both sexes, in the *realklasse* (one-year course, pupils 16 or 17); compulsory in the upper cycle for the science section (three-year course, pupils 16 to 18).

DOMINICAN REPUBLIC. *Aritmética razonada y álgebra* (1st year); *geometría plana* (2nd year); *geometría del espacio y trigonometría* (3rd year); *elementos de matemáticas superiores, geometría y trigonometría, dibujo lineal* (4th year, physics and mathematics section). Compulsory in the first three years, and in the 4th year for the physics and mathematics section (four-year course, pupils 15 to 18).

ECUADOR. *Aritmética y geometría* (1st and 2nd years); *álgebra y geometría* (subsequent years). Compulsory throughout (six-year course, pupils 12 to 18).

EGYPT. Arithmetic (first four years); geometry (throughout course); algebra (in and after the 3rd year); trigonometry and mechanics (6th and 7th years); analytical geometry (7th year). Compulsory throughout the lower cycle (four-year course, pupils 10 to 14), and for all pupils in 1st year, and for science section pupils in 2nd and 3rd years, of the upper cycle (three-year course, pupils 14 to 17).

FINLAND. Arithmetic (1st, 2nd, and 3rd years); algebra (2nd to 8th year inclusive); trigonometry (8th year). Compulsory in all years of all sections (eight-year course, pupils begin at 11).

FRANCE. *Mathématiques.* Compulsory in all years of all sections (seven-year course, pupils 11 to 18). Optional, supplementary teaching in mathematics is offered in some sections.

GERMAN FEDERAL REPUBLIC. *Mathematik-Unterricht* or *Rechnung-Unterricht.* Compulsory throughout, with some exceptions (nine-year course, pupils 13 to 19).

GREECE. Practical arithmetic and geometry (1st and 2nd years); theoretical algebra and geometry (3rd, 4th, and 5th years); theoretical trigonometry and geometry (6th year). Compulsory throughout (six-year course, pupils 12 to 18).

GUATEMALA. *Matemáticas.* Compulsory in first four years (five-year course, pupils begin at 12 or 13).

HAITI. Algebra (all years); geometry (in and after the 2nd year); arithmetic (1st to 4th years inclusive); trigonometry (in and after the 5th year); projective geometry (6th year); algebra, complementary geometry, descriptive geometry, theoretical arithmetic, cosmography (7th year). Compulsory in the first six years in all sections, and in the 7th year for science-languages section.

HONDURAS. *Aritmética prática* (1st and 2nd years); *álgebra y geometría* (3rd and 4th years); *trigonometría* (5th year). Compulsory throughout (five-year course, pupils 13 to 17).

HUNGARY. Mathematics. Compulsory throughout (four-year course, pupils 14 to 18).

INDIA. Arithmetic, algebra, geometry; also in some cases in upper years elementary mechanics and trigonometry. Compulsory throughout for boys; arithmetic compulsory and geometry and algebra optional, for girls (five-year or six-year course, pupils 11 to 16 or 17).

IRAQ. Arithmetic, geometry (1st year); algebra (in and after 2nd year); trigonometry (in and after 4th year). Compulsory throughout (five-year course, pupils 12 to 19).

IRELAND. Arithmetic, algebra, geometry (including trigonometry). Compulsory in the first four years, up to the Intermediate Certificate (pupils 12 to 15 or 16), optional in the last two years (pupils 16 to 18).

ISRAEL. Algebra, geometry (1st and 2nd years); trigonometry, solid geometry (3rd and 4th years); trigonometry, solid geometry, coordinate geometry, calculus (3rd and 4th years, mathematics-physics section). Compulsory throughout (four-year course, pupils 14 to 18).

ITALY. *Matematica.* Compulsory throughout (eight-year course in general secondary schools, seven-year course in teacher-training secondary schools, pupils 10 to 18).

JAPAN. Mathematics, applied mathematics. Compulsory in the lower cycle (three-year course, pupils 12 to 14), optional in the upper cycle (three-year or four-year course, pupils 15 to 17 or 18).

LAOS. Mathematics. Compulsory throughout (seven-year course, pupils 14 to 20). Optional, supplementary teaching in mathematics is offered in the classical section of the upper cycle.

LIBERIA. General mathematics, algebra (1st year); algebra, geometry (subsequent years). Compulsory throughout (four-year course, pupils 14 to 22).

LUXEMBURG. Arithmetic (1st and 2nd years); mathematics (subsequent years). Compulsory throughout, except that mathematics is not taken in the final year of the commercial section of the modern course.

MONACO. *Mathématiques.* Compulsory in all years in all sections (seven-year course, pupils 11 to 18). Optional, supplementary teaching in mathematics is offered in classical section B in the 5th and the 6th year.

NEPAL. Mathematics. Compulsory throughout (six-year course, pupils 11 to 16). Optional, supplementary teaching in mathematics is offered in the last two years.

NETHERLANDS. *Upper primary schools* (four-year course, pupils 12 to 16): optional. *Modern secondary schools* (five-year or six-year course, pupils 12 to 17 or 18): compulsory throughout, except that mathematics is not taken in the last two years of the modern "A" section (languages). *Grammar schools* (six-year course, pupils 12 to 18): compulsory throughout.

NEW ZEALAND. Core mathematics, mathematics, mechanics, applied mathematics. Compulsory in the first three years, i.e., up to the School Certificate Examination; optional in the 4th and the 5th year (five-year course, pupils 13 to 18).

NICARAGUA. *Aritmética razonada y comercial* (1st year); *álgebra* (2nd year); *geometría plana y del espacio* (3rd year); *trigonometría, nociones de topografía* (4th year); *revisión de matemáticas* (5th year). Compulsory throughout (five-year course, pupils 13 to 18).

NORWAY. *Modern secondary schools:* arithmetic, algebra, geometry, applied arithmetic (1st and 2nd years); social mathematics (3rd year). *Grammar schools:* the designation of the subject varies according to the year (see report). *Modern secondary schools* (three-year course, pupils 14 to 17): compulsory throughout. *Grammar schools* (five-year course, pupils 14 to 19): mathematics and natural science sections, compulsory throughout; linguistic sections, compulsory throughout, except in the final year, when mathematics is not taken.

PAKISTAN. East Bengal. Arithmetic, geometry (1st year); arithmetic, algebra, geometry (subsequent years). Compulsory throughout (five-year course, pupils 11 to 15). Optional, supplementary teaching in mathematics is offered in the 4th and the 5th year.

PANAMA. *Matemáticas.* Compulsory in first two years, optional thereafter (six-year course, pupils 12 to 20).

PERU. *Aritmética* (1st year); *álgebra* (2nd and 3rd years); *geometría plana* (4th year); *geometría del espacio y nociones de trigonometría* (5th year). Compulsory throughout (five-year course, pupils 12 to 16 or, in some cases, 18, 19 or 20).

PHILIPPINES. General mathematics (2nd year); advanced arithmetic (3rd year); advanced algebra and geometry (3rd year, optional courses). Compulsory in 2nd and 3rd years (four-year course).

POLAND. Mathematics. Compulsory throughout (four-year course, pupils 14 to 18).

PORTUGAL. Mathematics. Compulsory throughout the first five years, and in four sections in the last two years (seven-year course, pupils 10 to 17).

ROUMANIA. Arithmetic, algebra, geometry, trigonometry. Compulsory throughout (six-year course, pupils 11 to 17). Optional, supplementary work in mathematics is done in the school mathematics societies.

SALVADOR. *Matemáticas - aritmética y geometría* (1st year); *matemáticas - álgebra y geometría* (2nd year); *geometría intuitive, álgebra y trigonometría* (3rd year); *geometría plana y trigonometría* (4th year); *geometría del espacio y cosmografía* (5th year). Compulsory throughout (five-year course, pupils 13 to 17).

SPAIN. *Aritmética y geometría* (1st and 2nd years); *aritmética, álgebra y geometría* (3rd year); *álgebra y geometría* (4th year); *análisis matemáticas, complementos de geometría, trigonometría* (5th year); *análisis matemática* (6th year). Compulsory throughout lower cycle (four-year course, pupils 10 to 14), and in the science section in the upper cycle (two-year course, pupils 14 to 16).

SWEDEN. Mathematics. Compulsory in (a) all years in the modern secondary schools (four-year or five-year course, pupils 11 to 15, or 13 to 16); (b) grammar schools (pupils 16 to 19), in all years of the *real* section and the modern section (social science option), and in the 1st year of the modern languages and the Latin sections; (c) girls' secondary schools, except in the final year of the practical sections.

SYRIA. Arithmetic, elementary algebra (1st and 2nd years); algebra, plane geometry (3rd and 4th years); solid geometry, algebra (5th year); elementary infinitesimal calculus, trigonometry, theoretical arithmetic (6th year); mechanics, descriptive astronomy, conic sections, descriptive geometry (7th year). Compulsory throughout (seven-year course, age of admission 10-15 (lower cycle) or 14-20 (upper cycle).

THAILAND. Arithmetic, algebra, geometry. Compulsory throughout (two three-year cycles).

UKRAINE. Arithmetic (1st year); arithmetic, algebra, geometry (2nd year); algebra, geometry, technical drawing (3rd and 4th years); algebra, geometry, trigonometry, technical drawing (5th and 6th years). Compulsory throughout (six-year course, pupils 11 to 17).

UNION OF BURMA. Arithmetic (first three years); elementary algebra and geometry (in and after 1st year); trigonometry (in and after 4th year). Compulsory throughout (five-year course, pupils begin at 11).

UNION OF SOUTH AFRICA. Geometry, algebra (first three years); geometry, trigonometry, algebra, graphs (subsequent years). Compulsory only for pupils preparing for study at university level (five-year course, pupils 14 to 18).

U.S.S.R. Arithmetic (1st and 2nd years); algebra, geometry (2nd to 6th years inclusive); trigonometry (5th and 6th years). Compulsory throughout (six-year course, pupils 11 to 17). Optional, supplementary work in mathematics is done outside class hours.

UNITED KINGDOM. (a) *England and Wales.* Mathematics. Almost invariably compulsory in the first five years, and optional in the last two years of the grammar schools (pupils 11 to 15, 16 or 18). — (b) *Northern Ireland.* Arithmetic, algebra, geometry, trigonometry (lower cycle); same subjects to a more advanced level, plus advanced mechanics (upper cycle). Compulsory throughout (pupils 11 to 18 in grammar schools, 11 to 15 in secondary intermediate schools, 13 to 16 in technical intermediate schools). The grammar schools also offer an optional advanced course. — (c) *Scotland.* Mathematics. Compulsory in first three years (three-year, five-year and six-year courses, pupils begin at 12), thereafter optional. There are easier syllabuses for girls and the less gifted pupils, and an advanced course for 6th year pupils preparing for the Scottish Leaving Certificate.

UNITED STATES. General mathematics or arithmetic (1st and 2nd years); algebra or general mathematics (3rd year); plane geometry (4th year); intermediate algebra (5th year); solid geometry, plane trigonometry (6th year). Compulsory during first three years, optional in subsequent years (six-year course, pupils 12-13 to 17-18).

VIETNAM. Arithmetic, algebra, plane geometry, geometrical drawing (2nd and 3rd years); algebra, solid geometry, geometrical drawing (4th year); algebra, plane geometry (5th year); algebra, trigonometry, solid geometry (6th year); arithmetic, algebra, trigonometry, mechanics, supplementary geometry, cosmography (7th year). Compulsory in and after the 2nd year (seven-year course, pupils 12 to 17 or 18).

NUMBER OF HOURS PER WEEK ALLOCATED TO MATHEMATICS

Apart from its status as required or non-required, the clearest indication of the importance attached to any given subject is undoubtedly the number of hours per week allocated to it, and the percentage this allocation represents of the total time-table.

In the light of that indication, mathematics in all cases ranks as at least one of the major subjects taken at secondary level, and as the major subject in the science sections.

The analysis of the table given below shows that in the majority of the countries the number of hours per week allocated to mathematics in all general secondary classes, except those of the science sections, is from four to six. A time allocation of three hours per week is considerably less frequent, and one of only two hours or less represents the exceptional case.

Expressed as percentages of the total time-table, these time allocations represent between 9% and 20% (25% in some cases). A percentage of less than 10% is rarely met with.

So far as the science sections to be found at upper secondary level are concerned, the number of hours per week allocated to mathematics is naturally even higher than in other classes, being from six to ten, or, in one country, twelve. Expressed as percentages of the total time-table, these figures represent from 20% to 40%, or 48% in the marginal case.

AFGHANISTAN. 1st to 5th years, 5 hours (14.28%). 6th year: mathematics section, 10 hours (28.56%); philosophy section, 3 hours.

AUSTRALIA. Classes taking mathematics as one unit, 4 to 6 hours (16.66% to 25%); classes taking mathematics as two units, 6 to 8 hours (25% to 33.33%); classes taking mathematics as three units, 10 hours (40%).

BELGIUM. Three-year lower cycle: Latin section, 4 hours (11.1%); modern section, 4 hours (12.5%). Three-year upper cycle, classical section: Latin-Greek sub-section, 3 hours; Latin-mathematics sub-sections, 7 hours; Latin-science sub-section, 5 hours. Three-year upper cycle, modern section: modern-scientific sub-section, 7 hours; modern-commercial sub-section, 3 hours.

BULGARIA. 1st, 2nd, and 3rd years, 5 hours (16.67%); 4th year, 4 hours (13.33%).

BYELORUSSIA. 1st to 6th years, 6 hours (20%).

CAMBODIA. 1st year, 2 hours (8%); 2nd year, 2½ hours (10%); 3rd and 4th years, 3 hours (12%); 5th and 6th years, 4 hours (15.4%); 7th year: mathematics class, 9 hours (32%); experimental science class, 4 hours; philosophy class, 1½ hours.

CANADA. Grades VII to IX, 12% to 16%; senior secondary grades, 12 to 20%, according to course chosen; grade XIII (Ontario and British Columbia), 36% for pupils intending to enter a science faculty.

CEYLON. Junior and senior cycles, 9 hours (20%).

CHILE. 1st year, 5 hours (16.12%); 2nd and 3rd years, 4 hours (10.76% and 10.81% respectively); 4th and 5th years, 3 hours (8.33%); 6th year: science section, 3 hours (8.10%); arts section, a total of 3 hours (8.10%) for mathematics and physics.

CHINA (REPUBLIC OF). Lower cycle, 3 hours; upper cycle, 4 hours (10%).

COLOMBIA. 1st and 2nd years, 5 hours (20.83%); 3rd year, 5 hours (21.73%); 4th year, 7 hours (28%); 5th year, 3 hours (12%).

COSTA RICA. 4 hours (11.11%) throughout.

CUBA. 1st and 2nd years, 5 hours (25%); 3rd and 4th years, 3 hours (15%); 5th year (science section only), 5 hours (25%).

CZECHOSLOVAKIA. 1st year, 7 hours (20%); 2nd year, 6 hours (18%); 3rd year, 5½ hours (17%); 4th, 5th, and 6th years, 6 hours (17.6%).

DENMARK. 1st year, 4 hours; 2nd year, 5 hours; 3rd and 4th years, 6 hours; 5th year *(realklasse)*, 5 hours (algebra and arithmetic) and 2 hours (geometry); 6th, 7th, and 8th years, 6 hours. (12.5% to 20%.)

DOMINICAN REPUBLIC. 1st and 2nd years, 5 hours (20%); 3rd year, 4 hours (16%); 4th year (physics-mathematics section), 12 hours (48%).

ECUADOR. 1st and 2nd years, 5 hours (15.62%); 3rd and 4th years, 4 hours (12.5%); 5th year, 3 hours (9.09%); 6th year, 3 hours (11.53%) and, for pupils taking mathematics option, 3 additional hours.

EGYPT. 1st to 4th years, 4 to 6 hours (11.1% to 16.7%); 5th year, 4 hours (11.1%); 6th and 7th years (science section only), 7 hours (18.4%).

FINLAND. 1st year, 5 hours; 2nd year, 3 hours; 3rd, 4th, and 5th years, 4 hours. 6th, 7th, and 8th years: mathematics section, 5 hours; languages section, 3 hours.

FRANCE. See report for details.

GERMAN FEDERAL REPUBLIC. Classical schools: 1st and 2nd years, 4 to 5 hours; 3rd year, 4 hours; 4th and 5th years, 3 to 4 hours; 6th, 7th, 8th, and 9th years, 3 hours. Modern schools: 1st, 2nd, and 3rd years, 4 to 5 hours; 4th, 5th, and 6th years, 3 to 4 hours; 7th and 8th years, 3 hours; 9th year, 3 to 4 hours. Scientific schools: 1st to 6th years, 4 to 5 hours; 7th, 8th, and 9th years, 3 to 5 hours.

GREECE. 1st year, 3 hours (8.5%); 2nd year, 4 hours (11%); 3rd year, 3 hours (8.5%); 4th year, 4 hours (10.5%); 5th and 6th years: classical section, 4 hours (10%); science section, 6 hours (18%) and 7 hours (20%) respectively.

GUATEMALA. 5 forty-minute lessons throughout.

HAITI. Section A: 1st and 2nd years, 5 hours; 3rd year, 4 hours; 4th and 5th years, 3 hours; 6th year, 4 hours. Section B: 1st and 2nd years, 5 hours; 3rd year, 6 hours; 4th, 5th, and 6th years, 5 hours. Section C: 1st and 2nd years, 5 hours; 3rd year, 6 hours; 4th, 5th, and 6th years, 5 hours; 7th year, 6 hours.

HONDURAS. 1st and 2nd years, 5 hours (13.88%); 3rd, 4th, and 5th years, 4 hours (11.76%, 9.30%, and 10% respectively).

HUNGARY. Classical section: 1st year, 5 hours (16.1%); 2nd year, 4 hours (12.5%); 3rd year, 4 hours (12.1%); 4th year, 4 hours (12.5%). Modern section: 1st year, 5 hours (16.1%); 2nd year, 6 hours (18.8%); 3rd year, 5 hours (15.2%); 4th year, 4 hours (12.5%).

INDIA. 1st, 2nd, and 3rd years, 6 forty-minute lessons (14.28%); 4th and 5th years and, where applicable, 6th year, 8 or 9 forty-minute lessons (20%).

IRAQ. 1st year, 7 hours (21.2%); 2nd and 3rd years, 6 hours (18.1%); 4th and 5th years, 5 hours (15.6%).

IRELAND. 5 hours (17% to 20%) throughout, or 3½ hours in the case of girls taking the elementary course.

ISRAEL. 4 hours (11% to 12.5%) throughout.

ITALY. 1st, 2nd, and 3rd years, 3 hours (12.8%). 4th year: science schools, 5 hours (21.7%); classical schools, 2 hours (8%); teacher-training schools, 4 hours (14.2%). 5th year: science schools, 4 hours (16%); classical schools, 2 hours (8%); teacher-training schools, 3 hours (10.3%). 6th year: science and classical schools, 3 hours (11.5%); teacher-training

schools, 2 hours (7.7%). 7th year: science schools, 3 hours (11.1%); classical schools, 2 hours (7.7%); teacher-training schools, 2 hours (8%). 8th year: science schools, 3 hours (10.7%); classical schools, 2 hours (7.4%).

JAPAN. 1st year, 4 to 5 hours (12% to 17%); 2nd and 3rd years, 3 to 5 hours (9% to 17%); upper cycle, a prescribed number of credits.

LAOS. Gradual increase from 2 hours in the 1st year to 9 hours in the mathematics class in the 7th year.

LIBERIA. 3¾ hours throughout.

LUXEMBURG. See report for details.

MONACO. See report for details.

NEPAL. 4½ hours (13-14%) throughout.

NETHERLANDS. See report for details.

NEW ZEALAND. 1st year, 2 or 3 forty-minute lessons; 2nd year, 6 or 7 forty-minute lessons; 3rd and 4th years, 7 forty-minute lessons.

NICARAGUA. 1st, 2nd, and 3rd years, 5 hours (16.66%); 4th and 5th years, 3 hours (10%).

NORWAY. *Modern secondary schools.* 1st and 2nd years, 5 hours (16.12%); 3rd year, 4 hours (12.5%). *Grammar schools.* 1st and 2nd years, 5 hours (16.12%). 3rd year: mathematics section, 6 hours (20%); natural science and linguistic sections, 4 hours (12.5%). 4th year: mathematics section, 6 hours (20%); natural science and linguistic sections, 5 hours (16.12%). 5th year: mathematics section, 8 hours (28.59%); natural science section, 5 hours (16.12%).

PAKISTAN. *East Bengal.* 6 forty-minute lessons (15.4%) throughout.

PANAMA. 1st year, 4 hours (12.5%); 2nd year, 4 hours (13.7%); 3rd year, 4 hours (12.12%); 4th, 5th, and 6th years, 5 hours.

PERU. 4 hours (13.3%) throughout.

PHILIPPINES. 2nd year, 3½ hours (12.5%); 3rd year, 3½ hours (12.5%) during one semester.

POLAND. 1st year, 5 hours (14.7%); 2nd year, 4 hours (10.3%); 3rd and 4th years, 4 hours (11.8%).

PORTUGAL. 1st and 2nd years, 3 hours (15%); 3rd, 4th, and 5th years, 3 hours (12.5%); 6th and 7th years, sections c, f, g, and h only, 4 hours (19%, 20%, or 23.5% according to section).

ROUMANIA. 1st year, 6 hours (20%); 2nd year, 6 hours (18.8%); 3rd year, 5 hours (15.7%); 4th, 5th, and 6th years, 6 or 7 hours (17% or 25%).

SALVADOR. Three-year lower cycle, 5 hours; two-year upper cycle, 6 hours.

SPAIN. 1st to 4th years, 3 hours (15%); 5th year, science section only, 6 hours (25%); 6th year, science section only, 3 hours (15%).

SWEDEN. See report for details.

SWITZERLAND. See report for details.

SYRIA. Four-year lower cycle, 4 hours (14.28%). Three-year upper cycle : arts section, 3 hours (10.71%) ; science section, 6 hours (21.42%).

THAILAND. Three-year lower cycle, 4 hours (13.33%) ; three-year upper cycle, 6 hours (20%).

UKRAINE. 6 or 7 hours (20%).

UNION OF SOUTH AFRICA. 1st year, 1¼ hours (5%) ; 2nd and 3rd years, 4 hours (16.6%) ; 4th and 5th years, 5 hours (20%).

U.S.S.R. 6 hours (20%) throughout.

UNITED KINGDOM. *England and Wales:* 12.5% to 14.3%. — *Northern Ireland:* grammar, secondary intermediate, and technical intermediate schools : an average of 5 hours (roughly 20%, 15%, and 20% respectively). — *Scotland:* 1st, 2nd, and 3rd years, usually 6 forty-minute lessons, or 4 forty-minute periods for short course ; 4th and 5th years, 7 periods.

UNITED STATES. 1st and 2nd years, 5 periods (16.66%) ; 3rd year, 5 periods (20% to 25%) ; 4th, 5th, and 6th years, 5 periods (25%). (A period lasts 45-55 minutes.)

VIETNAM. 2nd year, 3 hours (10.34%) ; 3rd and 4th years, 3½ hours (12%) ; 5th and 6th years (mathematics section), 6 hours (20.68%) ; 7th year (mathematics section), 8 hours (27.5%).

YUGOSLAVIA. Slovenia : 9.4% to 15.6% ; Macedonia : 9.4% to 13.3%.

II. AIMS OF MATHEMATICS TEACHING

The aims of mathematics teaching at secondary level are expressed in a variety of ways, but may nevertheless be grouped under the following four heads : (a) educational aims, (b) general education aims, (c) practical aims, and (d) preparation for scientific or technical studies at higher level.

Educational aims. These aims relate to character formation on the one hand, and to the development of intellectual capacity on the other. In about half the countries mathematics is regarded explicitly as a subject which calls for great concentration, observation, exactitude, and perseverance, and therefore as of the highest importance from an educational point of view, since its study inculcates industry and imparts a mental discipline and an objective attitude. In two thirds of the countries, moreover, mathematics is specifically considered a means of cultivating aptitudes such as the power of abstraction, precision in the use of words, logical thinking, the analytical and the research attitude, judgment, and a scientific approach. The aims of mathematics teaching, in short, are the cultivation and ripening of moral and intellectual faculties contributing to the development of personality.

General education aims. Although only one third of the replies explicitly say so, mathematics obviously forms part of a general education, and its aims at secondary level are therefore in part those of secondary education as a whole. Mathematics ranks as a major subject in secondary

curricula because it acts as a source of cultural enrichment in a variety of ways : it contributes to the understanding of other subjects, and of the quantitative aspects of all human activity (production, consumption, commerce, etc.) ; it engenders appreciation of the value of the exact sciences and of the part they have played in man's evolution, and awareness of the laws governing the physical world ; in affording practice in abstract thought, it imparts one of the hallmarks of sound education, the ability to express general concepts in symbolic form.

Practical aims. Two thirds of the countries participating in the present inquiry refer to the practical aims of mathematics teaching at secondary level. Most persons now need a certain minimum of mathematical knowledge for the solution of their everyday problems of living, and must acquire it in the course of their secondary education. They also need some mathematical knowledge for an increasing number of professional activities. It is no longer only the specialists who require skill in calculation, ability to interpret graphs and statistical data, and understanding of economic problems. Some of the countries go so far as to state that it is the duty of every citizen to possess the minimum of mathematical knowledge enabling him to participate actively in his country's economic affairs.

Preparation for studies at higher level. One of the main aims of mathematics teaching at secondary level is to lay a sound foundation of mathematical knowledge for pupils intending to follow the scientific or technical studies at higher level which are called for by an increasing number of professions.

AFGHANISTAN. Scientific attitude, logical thinking, and interest in research.

AUSTRALIA. Capacity to interpret environment from a quantitative point of view and to deal with quantitative aspects of social and economic questions ; preparation for a profession.

BELGIUM. Logical thinking, creative imagination, observation, and analysis ; scientific training ; working skills, and ability to deal with mathematical questions arising in professional work or everyday life.

BULGARIA. Knowledge of definite use in everyday life ; preparation for higher studies.

BYELORUSSIA. Preparation for study of all sciences ; training in accuracy of speech, concentration, orderliness, and perseverance ; formation of judgment.

CAMBODIA. Mathematical education ; clarity, exactitude, and logical thought ; preparation for scientific studies at higher level.

CANADA. Statements of aims are made by seven of the ten provinces. Ability to apply mathematics to solution of everyday problems and to think logically and critically ; awareness of importance of accuracy ; in senior grades : preparation for university studies or for use of mathematics in commercial or technical fields.

CHILE. Capacity to analyse and interpret problems, deductive and logical thinking, accuracy in the use of words, precision, and perseverance ; aims of secondary education as a whole.

China (Republic of). Understanding of the general principles of mathematics and the relationships between the various aspects, acquisition of knowledge relating to daily life, development of deductive reasoning, analytical ability and aptitude for research, preparation for further study.

Colombia. Character training (reflection, concentration, exactitude, objectivity); intellectual training (reasoning power); cultural development; knowledge of practical value.

Costa Rica. Research attitude and power of analysis.

Cuba. Knowledge of practical value.

Czechoslovakia. General education; basic mathematical knowledge; polytechnical education; logical thinking and power of abstraction; preparation for studies at higher level.

Denmark. Logical thinking and ability to calculate with confidence and facility, and to express conclusions briefly and accurately; in upper years: rigorous reasoning and accuracy in use of words.

Dominican Republic. General education; preparation for studies at higher level; mental habits favouring development of personality.

Ecuador. The elementary mathematical knowledge demanded by everyday life; logical thinking and accuracy in use of words.

Egypt. In lower cycle: knowledge and skills of practical value; in upper cycle: introduction to scientific modes of reasoning.

Finland. Intellectual power to think logically, clearly, accurately and objectively, to analyse the factors of a problem, to represent natural phenomena in quantitative terms, and to work in accordance with a plan; knowledge and skills necessary for solution of problems; preparation for studies at higher level.

France. Aims of secondary education as a whole, including the formation of an élite class.

German Federal Republic. Intuition, accuracy of expression, ability to concentrate and to work conscientiously and methodically, reasoning power, and comprehension of mathematical relationships; knowledge and skills of practical value.

Greece. Aims of secondary education as a whole, including development of character, training in scientific and independent thought and preparation for future tasks.

Guatemala. Understanding of quantitative aspects of human activities (production, consumption, labour, etc.) and of space and time; mental discipline (accuracy, speed, etc.); awareness of practical realities.

Haiti. General education; disciplined reasoning and judgment; knowledge necessary for study of other science subjects; preparation for studies at higher level.

Honduras. Knowledge and skills necessary for everyday life; logical thinking and awareness of social significance of mathematics.

Hungary. Capacity to understand and formulate the laws governing the environment, to represent spatial relationships graphically, and to think clearly and systematically.

INDIA. Power to grasp the mathematical aspects of everyday life; training in reasoning, abstraction, and judgment; preparation for studies at higher level.

IRAQ. Logical thinking, and accurate and rapid solution of problems.

IRELAND. Reasoning power; accuracy of thought and expression; general education.

ISRAEL. Imparting mathematical techniques for solving practical problems; power to reason logically; preparation for studies at higher level.

ITALY. At lower secondary level: ability to calculate rapidly and with reference to practical problems; at upper secondary level: logical thinking, orderliness, and exactness and soberness in use of words; mental development and discipline; for teacher trainees: instruction in methodology of mathematics.

JAPAN. In lower cycle: understanding of practical value of mathematics in personal and social affairs, and in any situation requiring informed thinking and accurate diagnosis; in the upper cycle: comprehension of the basic concepts, laws, and principles of mathematics, and of capacity to apply them and to present data, reasoning, and conclusions accurately.

LAOS. Scientific training; introduction to abstract reasoning; preparation for scientific studies at higher level.

LIBERIA. In 1st year: basic knowledge and skills, and speed and accuracy; in subsequent years: good habits of working, orderliness, perseverance, and honesty.

LUXEMBURG. Logical attitude, intuition, and aptitude for research; essential mathematical knowledge.

MONACO. Intellectual training; preparation for specialist studies at higher level.

NEPAL. Basic mathematical knowledge; preparation for technical studies at higher level.

NEW ZEALAND. Knowledge and skills necessary for everyday life; preparation for studies at higher level; general education.

NICARAGUA. Aims of secondary education as a whole, including all-round development of aptitudes, and preparation for studies at higher level.

NORWAY. In modern secondary schools and lower years of grammar schools: basic knowledge and skills, including solution of practical problems; in upper years of grammar schools: preparation for practical applications and research (science section); power to solve practical problems and to comprehend economic relationships (linguistic sections).

PAKISTAN. *East Bengal.* Understanding of the practical value of mathematics in individual, social, and economic affairs; inculcating certain habits, interests, and skills; general education.

PANAMA. General education; character training; power to think and act with clarity and perseverance, and to reason logically; basic mathematical knowledge and skills, and ability to apply mathematical symbols.

PERU. In 1st year : completing and consolidating knowledge gained at primary level, and preparation for more advanced work ; in subsequent years : scientific aptitude and reasoning power.

PHILIPPINES. Ability to solve practical problems and to think logically, quantitatively, and critically ; awareness of the value of mathematics as a cultural heritage and as a factor in human progress; developing character traits such as honesty, perseverance, and accuracy.

POLAND. Knowledge which serves to interpret reality (economic problems, for example) ; instructional aims ; educational aims.

PORTUGAL. Knowledge ; reasoning power, precision of thought and expression, observation, initiative, clarity, speed, and good habits of working ; preparation for studies at higher level.

ROUMANIA. Capacity to think logically, to reflect, to concentrate, and to generalize ; accuracy ; awareness of order and beauty ; preparation for study of other subjects and for meeting problems of everyday life.

SALVADOR. In the lower cycle : understanding of mathematical relationships ; individual aptitudes ; observation, research, criticism, induction, concise and accurate use of words ; application of mathematical reasoning to human activities and to solution of everyday problems ; in the upper cycle : power to think abstractly, logically, and independently, and to apply mathematical knowledge in everyday life.

SPAIN. Developing intelligence through logical thought ; essential minimum of mathematical knowledge ; preparation for studies at higher level.

SWEDEN. Lower and girls' secondary schools : ability to apply mathematics to practical problems ; upper secondary schools : extending and deepening mathematical knowledge ; introduction to mathematical modes of thought.

SWITZERLAND. *Geneva.* Imparting of knowledge and skills, development of intellectual aptitudes (scientific spirit, etc.) and moral capacities (honesty, conscientiousness, etc.). — *Ticino.* General education and preparation for higher studies.

SYRIA. Capacity to think ; accuracy ; preparation for studies at higher level.

THAILAND. Capacity for concentration, accuracy, rapidity, orderly thinking, confidence, etc.

UKRAINE. Mathematical groundwork ; understanding of quantitative aspects of real world and of space, orderliness, accuracy, concentration, and liking for individual work ; practical application ; preparation for studies at higher level and for free choice of an occupation.

UNION OF SOUTH AFRICA. Official instructions define the aims of mathematics teaching as imparting knowledge of a definite sequence naturally arranged, developing powers of reasoning and of working out mathematical truths.

U.S.S.R. Elementary knowledge for solution of practical problems ; logical thinking and comprehension of space ; general and polytechnical education ; preparation for an occupation and for studies at higher level.

UNITED KINGDOM. *England and Wales.* There are no official instructions concerning aims ; these are determined by the headmaster of each school. — *Northern Ireland.* There are no official instructions concerning aims ; pupils are required to acquire the requisite knowledge for admittance to university or technical studies. — *Scotland.* Aims given in " Junior Secondary Education " (published by Scottish Education Department) : capacity to apply mathematics to solution of practical problems ; preparation for an occupation ; observation and reasoning.

UNITED STATES. Comprehension of mathematical concepts, ease of calculation, awareness of the importance of mathematics, and power to express general ideas with symbols.

VIETNAM. General education ; reasoning power ; preparation for specialist studies at higher level.

YUGOSLAVIA. General education, development of intellectual faculties (reasoning, abstraction, etc.) and moral attitudes (orderliness, accuracy, perseverance, etc.), practical utility, preparation for higher studies.

III. SYLLABUSES

Details as to the extent and main content of the secondary mathematics syllabuses will be found in each of the reports on the countries participating in the present inquiry. A comparative study of these syllabuses would necessarily go far beyond the bounds of the present inquiry, limited as it is to a general survey of the problems involved in mathematics teaching.

Comment must therefore be restricted to the modifications made recently or impending in secondary mathematics syllabuses in about half the countries, as indicated in the individual reports in the second part of the volume.

The reasons given for these modifications are in some cases pedagogical ones. They clearly reveal the desire to bring secondary mathematics teaching into line with modern educational ideas and effective pupil needs and intellectual capacity by placing emphasis on the understanding of concepts rather than on the acquisition of mechanical skills, by giving more practical exercises and applications, and by the introduction of direct methods, up-to-date teaching aids, and alternative syllabuses offering above-average pupils opportunities of doing more advanced work.

In other cases the modifications are ascribed to the influence of modern developments in the mathematical field. There have been considerable differences in the time lag, but the overwhelming importance assumed by scientific research and its applications in industry has inevitably affected the training offered the pupils, those in science sections in particular, who will take up a technical career.

The question as to what precise extent modern developments in the mathematical field have affected the secondary teaching of mathematics was answered by some twenty of the countries. In some cases the reply states merely that those developments were taken account of in the formulation of the secondary mathematics syllabuses. In other cases details are given of definite modifications made or impending in those

syllabuses, including the introduction of infinitesimal calculus, coordinate geometry, statistics, etc., and added stress on functions, vectors, the calculation of probability, differential and integral calculus, and applied mathematics.

IV. METHODS OF TEACHING MATHEMATICS

Directions on Methods

Official Instructions. In reply to the question the purpose of which was to show whether secondary mathematics teachers are obliged or not to teach on lines laid down by the education authorities, over half the countries participating in the inquiry state that there are official instructions on methods accompanying the syllabuses. The replies or the instructions themselves make it clear that the latter are in no way really obligatory, but are more in the nature of suggestions or advice, in many cases of a very general kind, which are based on certain educational or psychological principles, and which the teacher is free to adopt, with suitable adaptation to the particular circumstances and intellectual level of his class, or reject.

Although few of the countries refer explicitly to the point, it is evident that one of the main tasks of inspectors is to give teachers advice on methods. In the United States, the teachers' own professional organizations play a similar role.

About ten of the countries state that teachers receive instruction in methodology during their period of teacher training, and may therefore be expected to employ the methods they deem the most appropriate.

General Directives. Whether given official embodiment or not, the following methodological principles of secondary mathematics teaching occur and recur in the replies :

(a) awakening the pupils' interest, adapting teaching to their degree of intellectual maturity, proceeding from the concrete to the abstract, avoiding mere memorization and formalism, encouraging pupil participation ;

(b) giving priority to intuition and experimentation in the lower years, especially in regard to geometry, and then gradually introducing logical and deductive reasoning and individual research ;

(c) gradually leading pupils to the point where they can themselves discover laws, and where greater emphasis can be placed on reasoning than on automatic procedures ;

(d) illustrating teaching with numerous practical exercises (measuring, surveying, making teaching materials, etc.) ; in the countries of Eastern Europe the exercises and practical applications given are based on the concepts of polytechnical education.

Correlation of Mathematics with Other Subjects. Roughly three quarters of the countries state that at secondary level the teaching of mathematics is correlated, incidentally or in routine fashion, with that of other subjects, physics and chemistry in particular, geography and the social sciences to a lesser extent. The reply from France states in this connection that the teaching of mathematics is coordinated with the teaching even of such

subjects as the mother tongue (precision in the use of words) and history (the history of scientific thought).

Few of the countries give details as to how such correlation is established. One of the commonest procedures appears to be meetings of the various specialist teachers to arrange that the mathematical knowledge required for understanding certain topics, say, in physics, shall be taught at a suitable time in advance. The correlation of mathematics with another subject is facilitated when one and the same teacher takes both of them.

The following table summarizes the position in the various countries in regard to the methodology of mathematics teaching at secondary level.

AFGHANISTAN. Methods the same as those used in French secondary schools; teachers are given instruction in them during their teacher training. Mathematics correlated with other subjects.

AUSTRALIA. No official instructions concerning teaching methods, only suggestions, are issued (appeal to pupils' interests, adaptation of work to individual capacity); most common method is instruction and demonstration by teacher, followed by individual work on exercises; less frequently project and study assignment methods are used.

BELGIUM. Official recommendations: adaptation of syllabuses and methods to pupils' level; proceeding from the concrete to the abstract; in geometry, teaching to be intuitive at first, and then reasoning to be introduced gradually; frequent practical work. New methods based on recent discoveries in psychology. Mathematics correlated with other subjects, in particular physics (which may be taught by the same teacher). Practical activities.

BULGARIA. Official instructions: introductory talks, formal teaching. Mathematics constantly correlated with other subjects (especially in regard to chemistry and physics problems). Ample practical exercises, surveying in particular.

BYELORUSSIA. Recommendations concerning methods are issued by the Ministry of Education; methods employed depend on work in hand, and age and educational level of pupils (formal instruction by the teacher, or analysis by pupils themselves).

CAMBODIA. No official instructions. Mathematics correlated with physics, through teaching essential mathematical knowledge in advance. No practical applications.

CANADA. Syllabuses and textbooks contain instructions or suggestions concerning methods, but choice of methods is generally left to teacher; ordinarily explanation of the topic is followed by interrogation and discussion. Practical applications are included in syllabuses, and are of special importance in technical and commercial courses.

CEYLON. No official instructions concerning methods. Applied mathematics taught in senior classes.

CHILE. Suggestions concerning methods are contained in official syllabuses; emphasis on pupils' capacity for conscientious and independent work (activity approach).

China (Republic of). Official instructions to make instruction as comprehensive as possible; practical exercises, care taken to give pupils individual attention.

Colombia. Teachers free to choose methods; they generally seek to arouse pupils' interest, to lead them to elaborate ideas logically (induction, deduction) and to be objective. Mathematics correlated with physics. Practical applications.

Costa Rica. Official instructions: more emphasis to practical application than to theoretical proof; pupils to discover relationships and laws; avoiding mere memorization; presentation of problems by pupils.

Cuba. Official recommendations: appeal to intuition without neglecting demonstrations forming part of rational method. Mathematics correlated with other subjects, physics in particular, when syllabuses were formulated. Problems chosen which show practical applications of mathematics.

Czechoslovakia. Teachers' handbooks on methods accompany textbooks, and suggestions on methods are regularly given in journal "Mathematics at School"; achieving a balance between development of logic and acquisition of knowledge. Mathematics correlated with other subjects, especially physics. Practical applications (measuring, etc.) in open air, and in connection with technical drawing.

Denmark. No prescribed methods; methods used are those teachers have learned during teacher training. Mathematics correlated with other subjects, through teaching required mathematical knowledge in advance. Practical exercises frequently used on teacher's initiative, especially in lower classes.

Dominican Republic. Official recommendations: need to prevent intuition from being replaced entirely by deductive and inductive reasoning. Practical exercises.

Ecuador. Lower classes: concrete and objective teaching. Upper classes: generalization.

Egypt. Official instructions: teaching based on concrete teaching materials in first two years, and, in geometry, pupils should learn the proper use of their instruments; analytic and synthetic methods in 3rd and 4th years; inductive method in 5th year. Practical applications in 6th and 7th year (geodesy, use of theodolite).

Finland. Official instructions: explanations, examples, teaching as concrete and living as possible; plenty of practical exercises, mental calculations, and observations; practice in estimating degree of accuracy of approximate numbers. Mathematics correlated with physics. Applications to social phenomena.

France. Official instructions: activity methods, clarity, adaptation to intellectual level, no hasardous anticipatory explanations, no far-fetched proofs, periods of directed work. Mathematics correlated with physics, geography, and handicrafts, and also French (precision of speech) and history (history of scientific thought). Endeavour to start from level of children's experience. Surveying exercises.

GERMAN FEDERAL REPUBLIC. Methods chosen by teachers themselves; work generally introduced by discussion; emphasis given to intuition and experimentation (measuring) in lower classes. Mathematics correlated with physics (which is often taught by same teacher). Practical applications to problems in physics, technics, and economics. Practical work (plans, cardboard work, measuring).

GREECE. No official instructions. Application of modern educational ideas, in particular the principle of individual activity. Coordination with geography, biology, economics, and especially experimental physics. Some application exercises.

GUATEMALA. No official instructions concerning methods. Methods employed include deductive method, inductive method, and mixed (inductive-deductive) method. Mathematics correlated with physics and economics through choice of problems connected with these subjects. Exercises on construction and measurement (with a view both to teaching and to pupils' application of mathematics).

HAITI. No official prescriptions; exposition, explanation, directed work, grading of difficulties. Mathematics correlated with physics, through teaching required mathematical knowledge suitably in advance.

HONDURAS. Official suggestions: intuitive approach, group work, direct observation, concrete examples, etc. Visits to banks, undertakings, etc. Plenty of exercises.

HUNGARY. Official instructions: desirability of combatting formalism, coordinating theory with practice, pupil participation. Mathematics correlated with physics (teacher specializes in both subjects). Emphasis on practical exercises (surveying, etc.) in conformity with principles of polytechnical education.

INDIA. Official recommendations: simple, well graded examples; concrete problems. Mathematics correlated with other subjects, including physics, economics, geography, and social studies. Applied mathematics not generally taught; elementary mechanics taken in some states. Activity methods rarely used.

IRAQ. No official instructions; problem method commonly used. Mathematics correlated with other subjects.

IRELAND. No official instructions; methods employed are controlled by inspectors. Applied mathematics taught in last two years.

ISRAEL. No official instructions; choice of method depends on teacher's preferences and intellectual level of his pupils; frequently model lessons are arranged, followed by discussion. Mathematics correlated with other subjects (including physics and astronomy) through use of practical problems related to those subjects, and through teaching of mathematics and those subjects by same teacher. Practical work (constructions, measuring, etc.).

ITALY. Intuitive and experimental teaching in lower cycle, rational and systematic teaching in upper cycle.

JAPAN. Official recommendations: to develop comprehension by proceeding from the concrete to the abstract; to develop pupils' judgment through use of clear examples leading to mathematical accuracy and

precision ; activity method used mainly in geometry. Applied mathematics taught.

LAOS. Official suggestions : to understand pupils' difficulties. Mathematics correlated with other subjects, by agreement among teachers as to order in which topics shall be taught. Practical applications (geometrical and industrial drawing).

LIBERIA. No official instructions. Mathematics correlated with other science subjects. Practical work and applications.

LUXEMBURG. Official instructions : to proceed from the concrete to the abstract, to appeal to intuition, to use heuristic method.

MONACO. Official instructions : to proceed from the concrete to the abstract. Mathematics correlated with physics.

NEPAL. No official instructions ; general use of both theoretical and practical procedures ; introduction of direct method envisaged. Mathematics correlated with geography and science. Applied mathematics not taught.

NETHERLANDS. Euclidean method (adapted to pupils' level of understanding) ; intuitive method in some schools for beginning geometry.

NEW ZEALAND. No official instructions, but suggestions from lecturers and inspectors ; main principles : to begin each lesson with short mental work ; to restrict amount of new work introduced in a given lesson ; to get pupils working on problems as early as possible in lesson ; to give as much individual help as possible. Mathematics correlated with other subjects. Practical work in first two years, applied mathematics in remaining years for certain pupils (mainly those taking engineering courses).

NICARAGUA. No official instructions ; most teachers adopt theoretical and rational approach, based mainly on analysis, and give little attention to practical conclusions. Mathematics correlated with other subjects. Activity methods not in general use.

NORWAY. No official instructions ; all teachers are trained in methods ; heuristic and genetic method, pupil contribution ; experience and intuition in early stages of geometry course. Mathematics correlated with physics, economics, etc., through teaching of required mathematical knowledge in advance. Few practical exercises, apart from some measuring for testing calculations.

PAKISTAN. *East Bengal.* Official recommendations : analytic method in lower years, synthetic method subsequently. Mathematics incidentally correlated with geography, physics, astronomy, etc. Practical applications (mechanics, trigonometry, etc.) ; analytic method involves measurements, surveying, handicrafts, etc.

PANAMA. Official suggestions : adaptation to individual differences and interests, pupil participation, projects and directed work, individual correction of homework, avoidance of mechanical learning. Mathematics correlated incidentally with other subjects. Practical exercises.

PERU. Official instructions of a general kind : observation, demonstration through examples, analysis, synthesis ; teaching has definitely a practical bias ; activity methods (pupils make notes and record personal research in a *carpeta de trabajo*).

PHILIPPINES. Official suggestions : sequence of teaching is generally exploration, presentation (inductive and experimental approach), directed study, testing ; classes divided into groups. Mathematics correlated with physics, economics, etc., by choice of mathematical problems involving those subjects. Practical exercises precede formal teaching, and problems set derive from everyday life.

POLAND. Official suggestions (no given method is prescribed) : genetic method of induction (especially for geometry), experimentation, followed by discovery by pupils themselves of properties and laws (activity method). Mathematics correlated with natural science and physics. Practical work (problems, making models related to work done in class).

PORTUGAL. Official suggestions : coordination of mathematics with art and handicrafts in lower cycle, heuristic method subsequently. Mathematics correlated with other subjects, through collaboration among teachers (monthly staff meetings). Practical applications, especially in first two cycles (coordination with art and handicrafts, choice of practical problems).

ROUMANIA. Official instructions issued by Ministry of Education. Practical applications (measuring various areas, volumes, distances, heights, etc., topographical calculations, plan drawing).

SALVADOR. Official suggestions : projects method, adaptation to pupils' intellectual level, grading of difficulties, proceeding from concrete to abstract, proceeding inductively (in lower years), appeal to pupils' interests, emphasis on understanding rather than on automatic skills. Syllabuses make frequent reference to practical applications of mathematics ; teachers are urged to precede formal work with practical exercises.

SPAIN. Intuitive methods in beginning of course, then gradual introduction of logical reasoning and individual research.

SWEDEN. Official recommendations. Mathematics correlated with other subjects, through teaching required mathematical knowledge suitably in advance. Practical applications and exercises.

SWITZERLAND. *Geneva.* Methods left to the initiative of the teachers ; coordination with other subjects, particularly science ; practical applications through problems, little use made of activity methods. — *Ticino.* No official instructions ; coordination with other subjects left to the initiative of the teachers.

SYRIA. Official suggestions, and advice from inspectors. Mathematics correlated with other science subjects ; syllabuses take account of the interdependence of mathematics and those subjects. Little practical work.

THAILAND. Official suggestions : most teachers employ methods learned during their training. Mathematics correlated with experimental physics. Applied mathematics not taught as yet ; introduction of elementary statistics envisaged. Use of activity methods in experimental stage.

UKRAINE. Official recommendations covering a variety of methods ; teachers choose the most appropriate one ; no given method regarded as universally valid. Mathematics correlated with other subjects. Great variety of practical work (surveying, making teaching materials, analysis of information collected during visits to industrial and agricultural undertakings, etc.).

UNION OF SOUTH AFRICA. Syllabuses contain suggestions concerning methods ; the underlying principle is proceeding from the arithmetical concept to the algebraical and from the concrete to the abstract. Applied mathematics as such is not taught ; the setting of practical exercises is a matter for the initiative of the individual teacher.

U.S.S.R. Official instructions ; no given method is regarded as universally valid ; teaching must be systematic and adapted to the age of the pupils ; heuristic method and discussion method are commonly employed, especially in lower years ; formal exposition, followed by practical work, in upper years. Mathematics correlated with other subjects. Ample practical applications and exercises, in conformity with requirements of polytechnical education.

UNITED KINGDOM. *England and Wales*. Suggestions and adviceconcerning methods are given in a number of pamphlets which were published by the Board of Education. Practical work (surveying). — *Northern Ireland*. No official instructions or suggestions. Practical work non-existent except for advanced groups ; little use is made of activity methods except in the technical intermediate schools. — *Scotland*. Suggestions contained in official syllabuses ; activity methods warmly recommended, but not yet universally employed. Applied mathematics taught only to pupils taking additional course in dynamics.

UNITED STATES. No official instructions ; suggestions from professional associations ; generally exposition is followed by discussion and directed work ; trend towards induction. Mathematics correlated with physics, economics, etc., by appropriate choice of problems. Practical work preceding formal exposition (especially for low-ability pupils).

VIETNAM. Intuitive method. Mathematics correlated with physics, through teaching of required mathematical knowledge suitably in advance. Practical work (surveying, geometrical constructions, etc.).

YUGOSLAVIA. Official instructions in three republics. Inductive methods generally used ; deductive methods in upper classes. Coordination with other subjects, practical work.

THE TRANSITION FROM PRIMARY TO SECONDARY EDUCATION

On transfer from a primary to a secondary school many children experience difficulty in adapting themselves to their new surroundings and, above all, to new ways of working. This is a problem to which a number of educationists are now seeking a solution. Accustomed hitherto to being taught by a teacher responsible for the whole of their work, the children suddenly find themselves confronted by a group of specialist teachers, what is generally regarded as an overburdened curriculum, and a new rhythm of study.

In view of the importance of the problem, and of the fact that it is most acute in regard to the teaching of mathematics, it was deemed desirable for the present inquiry to include the collection of as much information as possible on any solution to it which the countries participating might have adopted.

The query made in this connection in the International Bureau of Education questionnaire, however, has elicited only a very limited amount of information. Half the countries have left the query unanswered, or simply state that there are no official instructions as to teaching methods for use in the transition from primary to secondary education. Other countries give hints among which only one definite indication is to be found, namely, that the continuity between primary and secondary education is ensured by revision at the beginning of secondary education of the work done in the primary school. This procedure appears to be fairly common, but even so it involves curriculum construction rather than methodology, and reveals a purely empirical approach to a problem that merits close study.

The few more definite indications given in the replies, which, while not offering any defined way of ensuring an untroubled transition from primary to secondary education, do at least throw some light on the problem and the psychological factors involved, include the following:

(a) special endeavours to assist pupils to pass from mechanical to reasoned procedures;

(b) concentration on the pupils' degree of mental readiness to go over from being taught in concrete terms to being taught in more abstract terms;

(c) greater emphasis on intuitive and concrete learning in the lower secondary years, and gradual transition to abstract reasoning;

(d) a slow and imperceptible change-over from intuition to rationality.

Other measures referred to in the replies include suggestions from the secondary teachers to the teachers of the top primary classes, the institution of a preparatory year in the secondary school, and the formation of homogeneous classes through grouping based on some form of selection, psychological or otherwise.

Textbooks and Teaching Materials

In about half the countries participating in the present inquiry, the choice of textbooks for secondary mathematics teaching is controlled more or less directly by the education authorities. Such control takes the following forms:

(a) A list of approved or recommended textbooks is published by the Ministry of Education, and the teacher's choice is limited to books appearing on this list. In many cases the examination and choice of textbooks are in the hands of a special committee.

(b) The teacher or the school must obtain the prior approval of the education authorities for the use of given textbooks.

(c) The education authorities refrain from recommending given textbooks, but lay down certain requirements (conformity with syllabuses, scientific standard, etc.) which textbooks must fulfil.

In some twelve of the countries the teachers are completely free to choose whatever textbook they deem most suited to the needs of their pupils, no official instructions being issued in this connection.

The inquiry has yielded little information in regard to the teaching materials and auxiliary aids used for mathematics lessons. About half the

countries refer to the usual materials such as geometrical models, measuring instruments, slide rules, tables, diagrams, etc., and some of these countries state that such materials should as far as possible be made by the pupils themselves. Auxiliary aids such as films, filmstrips, etc., have been brought into use in only about fifteen of the countries.

V. TEACHERS

Training of Mathematics Teachers

An inquiry was recently conducted on the training of secondary teachers, and in its published form [1] served as the working document for the XVIIth International Conference on Public Education. This Conference, moreover, drew up a recommendation (No. 38) on the subject to the Ministries of Education. The inquiry and the Conference discussions clearly revealed the vital importance of professional training for secondary teachers, in addition to their specialised studies ; a familiarity with teaching methods and adolescent psychology was recognised as being as necessary as a sound knowledge of the subject taught. This recognition of the need for professional as well as specialised training is confirmed by the details on methods given in the replies to the present inquiry ; the numerous recommendations on methodology clearly indicate that secondary teachers cannot rest content with a mere presentation of their subject, but need to avail themselves to the fullest extent possible of the resources offered by the educational sciences. In no less than three quarters of the countries participating in the present inquiry, moreover, secondary teachers of mathematics are required to have undergone professional training, that is to say, training of a general educational character. In cases where the details of this training are not given, reference may be made to the 1954 inquiry, already alluded to, on the training of secondary teachers in general.

As to their specialist qualifications, secondary teachers of mathematics are generally required to hold a first degree or a doctorate awarded by a faculty of physics or mathematics. In some cases, especially in certain Latin-American countries, their specialist studies for mathematics and their professional training is provided in one and the same institution, such as a higher school of teacher training or a faculty of education. In a few cases, secondary mathematics teachers have been trained only in a secondary school or as primary teachers.

The following table indicates the qualifications secondary teachers of mathematics are required to hold in each of the various countries :

Afghanistan. Baccalaureate at least required ; many teachers possess a first degree or a doctorate.

Australia. Basic teacher training, special training in the teaching of mathematics, and a degree with mathematics as part of course. Basic teacher training in the case of students appointed to teach mathematics (usually in lower classes) before completion of their university course.

[1] " Secondary Teacher Training " (Paris, Unesco ; Geneva, International Bureau of Education, Publication No. 155, 1954).

BELGIUM. Lower cycle : diploma of secondary teacher-training school (science section). All classes : first degree in mathematics and the *agrégé* secondary teacher's diploma.

BULGARIA. A pass in official examination concluding course of mathematical and physical sciences faculty (mathematics division) of State university.

BYELORUSSIA. Diploma awarded by physics-mathematics faculty of teacher training school or a university (four-year or five-year course), and completion of teaching practice.

CAMBODIA. Lower classes : primary teacher's diploma. Upper classes (French teachers) : first degree or a teaching diploma.

CANADA. Usually a university degree and not less than one year's professional training.

CEYLON. University degree, or teacher's diploma and experience.

CHILE. Title of *Profesor de Estado* awarded by institute of education in faculty of philosophy and education of University of Chile.

CHINA (REPUBLIC OF). Graduation from mathematics department of higher school of teacher training.

COLOMBIA. Baccalaureate (or teacher's diploma) and first degree awarded on completion of four-year specialist course in physics-mathematics faculty of Pedagogical University ; the course includes professional training.

COSTA RICA. Special training not organized ; preference given to candidates with engineering qualifications or institute of education diploma.

CUBA. Doctorate in science (preferably physics and mathematics).

CZECHOSLOVAKIA. *Grades VI, VII and VIII :* diploma of teacher training school (three-year post-secondary course). *Grades IX, X and XI :* diploma of higher school of pedagogy (four-year course) or a science faculty (five-year course). Academic and professional training (including practice teaching) ensured.

DENMARK. The title of *candidatus magister* (university course lasting five or six years, with mathematics as main subject and physics, chemistry and astronomy as auxiliary subjects) and six months' teacher training.

DOMINICAN REPUBLIC. Certificate of *Maestro normal de segunda enseñanza* with mathematics as special subject, or a university qualification.

EGYPT. Lower cycle : diploma of teacher training school (three-year post-baccalaureate course). Upper cycle : first degree in mathematics and institute of education diploma (one-year course), or diploma of higher school of teacher training (four-year course).

FINLAND. First degree in philosophy (covering mathematics, physics, and chemistry) and one-year's professional training.

FRANCE. Minimum requirement : first degree in mathematics teaching, and secondary teacher's certificate granted on basis of competitive examination covering theory and practice. Also recruitment at a higher scientific level for successful candidates in the *agrégation* examination.

GERMAN FEDERAL REPUBLIC. Four-year university course in mathematics and one other (science or arts) subject and two years' teacher training.

GREECE. First degree or doctorate awarded by the mathematics department of the faculty of physical and mathematical sciences.

GUATEMALA. Teacher training diploma of arts faculty of University of San Carlos and academic training at school of engineers. As this diploma was only recently established, many teachers do not have this qualification.

HAITI. Secondary teacher's certificate (three-year course at higher school of teacher training or (for supply teachers) science baccalaureate.

HONDURAS. Mathematics taught at secondary level by engineers, commercial experts, primary teachers. No arrangements for professional training as yet, but establishment of a higher school of teacher training is envisaged.

HUNGARY. Four-year university course with mathematics and physics as main subjects, and professional training.

INDIA. Generally, a degree in mathematics and a degree or diploma in education or teaching.

IRAQ. First degree awarded by higher school of teacher training or other higher establishment.

IRELAND. A first degree and post-graduate teacher training diploma.

ISRAEL. Doctorate (a first degree for lower classes) and teacher training.

JAPAN. University training (comprising at least a first degree).

LAOS. First degree or *agrégation* awarded by a university in France.

LIBERIA. B.A. or B.Sc. in education awarded by teacher training school of University of Liberia.

LUXEMBURG. Doctorate in physics and mathematics (four-year course) and two years' teaching experience.

MONACO. *Agrégation* and secondary teacher's certificate.

NEPAL. Degree or intermediate certificate from an Indian university (professional training not required).

NETHERLANDS. Upper primary schools : primary teacher's certificate in mathematics. Secondary schools : academic training or secondary teacher's certificate (official examination).

NEW ZEALAND. No official stipulations, but university studies in mathematics and one year's professional training are deemed desirable.

NICARAGUA. No special course of secondary teacher training. Mathematics is taught at secondary level by competent and experienced persons (holders of the baccalaureate and others). Establishment of a higher institute of education is envisaged.

NORWAY. Degree with mathematics as major or minor subject, and *cand. real.* or *cand. mag.* title awarded after completion of prescribed period of teacher training.

PAKISTAN. Degree in science and mathematics, and teacher training.

PANAMA. First-category teachers : secondary teacher's diploma with mathematics and physics as special subjects.

PERU. Four-year training at a national institute of education or university faculty of education, or civil engineering qualification.

PHILIPPINES. B.Sc. in education, with mathematics as special subject.

POLAND. Diploma of university or higher State school of education (five-year course) and first degree in mathematics.

PORTUGAL. First degree in mathematics (four-year course) and teacher's diploma (two-year course).

ROUMANIA. Academic and professional training at higher level in special four-year course of a physics-mathematics faculty or (for teachers of first three secondary years) two-year course at a teacher training institute, and a pass in State examination for university diploma of secondary teaching.

SALVADOR. Certificate from teacher training school and a pass in mathematics examination, or baccalaureate and a pass in an examination in mathematics and pedagogy, or an engineering diploma. Secondary teachers will in future be trained at recently established higher school of teacher training.

SPAIN. Full secondary teachers : first degree or doctorate in science and service as assistant secondary teacher.

SWEDEN. Modern secondary schools : title of *filosofie magister*. Grammar schools : title of *filosofie licentiat*. In either case such title must include two mathematics certificates.

SWITZERLAND. *Geneva.* First degree in mathematical science, supplemented by teaching certificate (university courses and teaching practice). — *Ticino.* Complete university studies, and secondary teacher's diploma.

SYRIA. Diploma in mathematics and teacher's certificate awarded by the Syrian University.

THAILAND. At least post-primary teacher's certificate, and preferably secondary teacher's certificate (five-year higher course) or a degree.

UKRAINE. Diploma awarded by physics-mathematics faculty (five-year course) or pedagogical institute (four-year course), including obligatory teaching practice.

UNION OF SOUTH AFRICA. B.Sc. (three-year course) and teacher's certificate (one-year post-graduate course at university or teacher training school).

U.S.S.R. Higher diploma awarded by physics-mathematics faculty (five-year course) or pedagogical institute (four-year course), including obligatory professional training.

UNITED KINGDOM. *England and Wales.* University degree or advanced qualifications for recognition as " qualified teachers ", professional training not compulsory, but encouraged. — *Northern Ireland. Grammar schools :* minimum qualification is degree with two progressive courses in mathematics ; professional training not essential. *Technical intermediate*

schools: degree with at least two courses in mathematics, and, if possible, scientific or industrial experience. *Secondary intermediate schools:* degree with two progressive courses in mathematics, or degree and teaching diploma, or three years' professional training. — *Scotland.* For specialist teachers, a first or second class honours degree in mathematics and a year's professional training in a teacher's college; for non-specialist teachers in lower classes, a university degree and a year's professional training.

UNITED STATES. Generally, bachelor's degree (four-year course with ordinary mathematics syllabus), a major degree for mathematics specialists.

VIETNAM. *Agrégation,* or first degree in mathematics plus teacher training, or diploma of higher school of education.

YUGOSLAVIA. Diploma of the university faculty of science and mathematics obtained after eight semesters of study ; pedagogical training necessary.

FACILITIES FOR FURTHER TRAINING

In practically all the countries participating in the present inquiry there are facilities for the further training of serving secondary teachers of mathematics. These facilities appear to be optional and in no way obligatory, with only a few exceptions (as in Roumania and Ukraine, for example, where secondary teachers of mathematics are required to attend a free refresher course once every five years).

Such facilities take the following forms :

(a) Further training courses. These are referred to by about half the countries, and are generally vacation courses organized by the education authorities themselves, the universities, the teacher training schools, or the teachers' professional associations.

(b) Lectures, seminars, conferences, meetings, etc., for specialist teachers. These are referred to by about thirty countries, and are usually held at regional or local level.

(c) Scholarships for further study at home or abroad, granted by the Ministries of Education, and other public, or private bodies. Scholarships of this kind are referred to by about fifteen of the countries.

(d) Specialist journals devoted entirely to mathematics or the teaching of mathematics, or containing occasional articles on such subjects. Publications of this kind are referred to by about twenty of the countries, and are often the organ of an association of mathematics teachers.

(e) Teacher exchanges. Only ten countries refer to this type of facility.

It is also of interest to note that the German Federal Republic and several countries in Eastern Europe have special institutes for the further training of teachers, in the form of courses, conferences, seminars, etc., and that in France secondary teachers may be seconded to a national scientific research centre for the purpose of doing scientific or teaching research.

The following table gives a summary of the further training facilities available in the various countries for secondary teachers of mathematics :

AFGHANISTAN. Scholarships.

AUSTRALIA. Teacher exchanges, refresher courses, seminars, a journal (" The Australian Mathematical Teacher ", published by the Mathematical Association).

BELGIUM. Teachers' meetings, advice of inspectors, conferences-meetings ; publications of the Belgian association of secondary mathematics teachers.

BYELORUSSIA. Regular further-training courses and seminars ; courses, conferences, and seminars organized at various administrative levels ; " pedagogical readings ".

BULGARIA. Courses, seminars, and workshops at the further training institute, summer courses, conferences, meetings arranged by the teachers' associations.

CANADA. Optional vacation courses at various universities, with allowances in some provinces for teachers attending.

CEYLON. Refresher courses in vacations.

CHILE. Further training courses throughout the country, circulars issued by the secondary education directorate ; school libraries, and official periodical *(Revista de Educación)* ; scholarships granted by the government or cultural bodies abroad.

CHINA (REPUBLIC OF). Associations of specialist teachers ; vacation seminars by the Taiwan provincial higher school of teacher training.

COLOMBIA. Scholarships, teacher exchanges.

COSTA RICA. A further training scheme for secondary teachers is now under examination.

CUBA. Scholarships for one year's study abroad.

CZECHOSLOVAKIA. Courses, conferences, workshops, and consultations organized by the regional further-training institutes ; periodical meetings at regional and district level, an official periodical (" Mathematics at School ").

DENMARK. Further training provided by secondary mathematics teachers' association in form of conferences, meetings, and reading circles receiving foreign periodicals ; international conferences.

DOMINICAN REPUBLIC. Teacher exchanges, lectures, special courses, seminars, study weeks, teachers' associations.

EGYPT. Further training seminars during school year and summer vacation, scholarships for study at home and abroad.

FINLAND. Summer courses organized by schools directorate in collaboration with University of Helsinki, courses and study days organized by the Finnish association of physics and mathematics teachers, a periodical published by this association, annual congress of teachers from Scandinavian countries.

FRANCE. Various associations of mathematics teachers, a periodical published by the association of mathematics teachers in public secondary schools, articles in *Cahiers pédagogiques pour l'enseignement du second degré*, discussions, lectures, regional meetings, allowances for conferences or

study abroad, possibility of doing research at national scientific research centre, various journals.

GERMAN FEDERAL REPUBLIC. Courses organized by further training institutes, bulletin published by *Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts*, lectures and conferences organized by this body.

GREECE. Two-year courses at teacher-training school, inspectors' advice, regional conferences, journals published by the Greek mathematics society, scholarships for study abroad.

GUATEMALA. Lectures, conferences, seminars, scholarships.

HAITI. Conferences organized by the national education directorate, scholarships.

HONDURAS. Special courses, vacation courses organized by secondary education directorate, teachers' meetings, study abroad.

HUNGARY. Courses and lectures organized by further training institutes, lectures organized by the Mathematics Society; books and periodicals.

INDIA. Refresher courses in some states, regional seminars, training college extension services.

IRAQ. Journal published by Ministry of Education.

IRELAND. Summer further training courses organized from time to time by Department of Education.

ISRAEL. Lectures, courses, model lessons organized by Ministry of Education and mathematics teachers' association journal.

JAPAN. Further training courses at teacher training schools and university, courses organized by local and regional bodies and educational associations.

LAOS. Periodicals *(Journal de mathématiques*, and *L'éducation mathématique)*.

LIBERIA. Vacation workshops.

LUXEMBURG. Books and periodicals from school and national library, government assistance for study abroad.

MONACO. Periodicals.

NEPAL. Endeavours are being made to improve existing facilities for further training.

NEW ZEALAND. Refresher courses organized by a special committee (financed by Ministry of Education) or university colleges.

NICARAGUA. Teachers' meetings to discuss teaching problems, scholarships for study abroad.

NORWAY. Meetings and vacation courses organized by secondary teachers' association, scholarships for study abroad, a periodical *(Nordisk Mathematisk Tidsskrift)*.

PAKISTAN. *East Bengal.* Refresher courses, teacher exchanges with countries abroad, journals, mathematics teachers' association.

PERU. Vacation courses, meetings.

PHILIPPINES. Periodicals and books, lectures, seminars, visits, study days, model lessons.

POLAND. Further training centre, teachers' associations, weekly and optional further training courses in large towns, annual vacation course attended by four hundred secondary mathematics teachers, a periodical (" Mathematics ").

PORTUGAL. Teacher exchanges, scholarships for further study at home or abroad, lectures, journals.

ROUMANIA. Further training institutes; once every five years mathematics teachers are required to attend a one-year refresher course covering individual study during the school year and courses and seminars occupying one month of the summer vacation; teachers' circles.

SALVADOR. Weekly meetings, correspondence courses, seminars, information on teaching questions, guidance and advice from inspectors.

SPAIN. Vacation courses, periodicals, scholarships for study abroad.

SWEDEN. State-subsidized refresher courses, other courses in preparation, lectures and study days organized by the association for the teaching of mathematics, a periodical *(Elementa)*.

SYRIA. Scholarships for study abroad, ministerial circulars, advice and guidance from inspectors.

SWITZERLAND. *Geneva.* Meetings, conferences and other activities of mathematics teachers, paid leave to attend conferences in Switzerland or abroad. — *Ticino.* Scholarships.

THAILAND. University refresher courses, correspondence courses organized by teachers' association.

UKRAINE. Compulsory attendance of free courses once every five years, regional educational conferences, consultations with specialists, model lessons given by outstanding teachers, teachers' committee in each school, lectures, seminars, meetings, books and periodicals.

UNION OF SOUTH AFRICA. Refresher courses held from time to time, meetings organized by the teachers' associations or convened by the inspectors.

U.S.S.R. Teachers' meetings in each school, regional pedagogical centres, annual conferences organized by further training institutes and Academy of Pedagogical Sciences, advice and guidance from experienced teachers, contacts through the secondary teachers' section of the Moscow Mathematics Society with leading Soviet mathematicians, a journal (" Mathematics at School "), ample literature.

UNITED KINGDOM. *England and Wales.* Refresher courses lasting one to ten days organized by Ministry of Education, the universities, and the local authorities. — *Northern Ireland.* Refresher courses organized by the Ministry of Education. — *Scotland.* Courses organized from time to time, some teacher exchanges, secondary mathematics teachers' associations, a periodical.

UNITED STATES. Teacher exchanges, courses, workshops, scholarships offered by industrial firms and professional bodies, annual national conferences, etc.

VIETNAM. Courses, advice and guidance from inspectors, periodicals.

YUGOSLAVIA. Courses, seminars and consultations organized by the association of teachers of mathematics and physics, periodicals and special publications, working groups in each school.

SUPPLY PROBLEMS

About half the countries reply affirmatively to the query in the International Bureau of Education questionnaire as to whether they experience difficulty in recruiting secondary mathematics teachers. It may be of interest to know the causes to which this difficulty is imputed.

The main reason most frequently advanced for this difficulty, and which certainly is one of almost universal validity, is the enormous technical development of the modern age. More and more specialists are being drawn into an ever-expanding industrial field which can offer science graduates considerably higher material rewards than the teaching profession.

The demand nowadays is undoubtedly for persons with scientific training, on the one hand, while it appears that enrolments in arts faculties are higher than those in science faculties, on the other. In explanation of this seeming paradox, certain of the countries, including Finland, Portugal, and the United States, state that the studies in science are more difficult and in some cases longer than the corresponding studies in arts, and that consequently a number of students are discouraged from taking up science.

Another reason for the difficulty of recruiting secondary mathematics teachers, advanced by only a few of the countries (Czechoslovakia, Poland, Syria, and Scotland in the United Kingdom) but nevertheless as cogent as the first, is the quantitative development of secondary education. The secondary school population is increasing so rapidly that it has practically outpaced school building capacity, for example, and plays a large part in the acute shortage of secondary mathematics teachers, and, as some of the countries point out, of secondary teachers as a whole.

Two isolated but significant comments in this connection in the replies merit special note. The reply from Greece states that the prevailing shortage of specialist secondary teachers is more acute in rural areas than in the towns, and ascribes this to the attraction of urban life. That from the United States ascribes the shortage in part to the relative lack of prestige, and the consequent relative unattractiveness, of the teaching profession.

The measures taken to overcome the difficulty of recruiting secondary mathematics teachers stem from its revealed causes, and include endeavours to improve the material rewards of secondary teaching, the offer of special facilities, scholarships in particular, in order to encourage students, science students especially, to take up the teaching profession, the re-appointment of retired teachers, assistance to primary teachers to enter the secondary field, the employment of foreign teachers, and, in one country, the exemption from military service.

AFGHANISTAN. No shortage.

AUSTRALIA. A general shortage, possibly more acute in regard to mathematics and science than in regard to other subjects. Remedies adopted : recruiting campaigns, financial assistance for teacher trainees, and for prospective teachers at secondary level.

BELGIUM. Shortage, due to attractive openings in engineering and other technical careers.

BULGARIA. No shortage.

BYELORUSSIA. No shortage.

CANADA. Shortage, due to better paid openings in industry. Remedies adopted : scholarships, allowances, increased salaries, improved pensions and conditions of work.

CEYLON. Shortage of mathematics teachers able to teach in the national languages, due to better paid openings outside teaching. Remedies adopted : mathematics made a required subject in the teacher training schools, and mathematics courses organized for in-service teachers without previous mathematical training.

CHINA (REPUBLIC OF). Recruitment difficult in rural districts owing to the increasing number of schools and the relatively small number of students specializing in mathematics. Non-specialist teachers with experience in this subject are sometimes employed.

COLOMBIA. Shortage of fully trained teachers, due to fact that such persons can obtain better paid posts outside teaching. Remedies : under consideration.

COSTA RICA. Shortage, due to insufficiency of teachers' salaries. Remedy : recent decision to raise teachers' salaries.

CUBA. No shortage.

CZECHOSLOVAKIA. Shortage, due to increase in number of schools and to attraction of careers in industry. Remedies adopted : the establishment of university correspondence courses for in-service primary teachers.

DENMARK. Shortage.

DOMINICAN REPUBLIC. Former shortage, overcome through establishment of higher normal schools with boarding accommodation, and scholarships covering all expenses.

EGYPT. Shortage, due to lack of mathematics graduates. Remedies adopted : creation of a higher normal school, due to open in June, 1956.

FINLAND. Shortage, due to lowness of teachers' salaries as compared with length (seven years) and severity of their university studies.

FRANCE. Shortage of all teachers required to hold high scientific qualifications, due to better paid posts in industry. Remedies envisaged : suitable counselling for secondary pupils.

GERMAN FEDERAL REPUBLIC. General shortage of secondary teachers, due to better paid openings in industry, commerce, etc.

GREECE. Shortage, due to lowness of teachers' salaries, and attraction of life in larger towns. Remedies adopted : improvement of teachers' salaries and added facilities for promotion.

HAITI. General shortage of secondary teachers, due to lowness of salaries. Remedies envisaged : improvement of teachers' salaries.

HUNGARY. No shortage.

INDIA. Shortage only in respect of women teachers.

IRELAND. Shortage, due to fact that comparatively few university students choose to study mathematics, and that a good proportion of these few prefer to enter industry or vocational teaching.

ISRAEL. No shortage.

JAPAN. Shortage. Remedies adopted : correspondence courses, and lectures.

LAOS. Shortage of mathematics teachers, but less acute than that of physics teachers.

LIBERIA. Shortage, due to former insufficiency of scholarships and teachers' salaries. Remedies adopted : sufficient scholarships are now available, and teachers' salaries have been improved.

LUXEMBURG. No shortage ; supply of mathematics teachers exceeds the demand.

MONACO. No shortage.

NEPAL. No shortage.

NETHERLANDS. Shortage of qualified teachers.

NEW ZEALAND. Acute shortage. Remedies adopted : increase of scholarships for teacher-trainees.

NICARAGUA. Shortage of trained teachers.

NORWAY. Shortage, due to better paid openings in research, industry, and insurance.

PAKISTAN. No shortage.

PANAMA. No shortage.

PERU. No shortage.

PHILIPPINES. Shortage, due to lack of mathematics specialists following decrease in demand for them.

POLAND. Shortage, due to increase in enrolments and demand for specialists for an expanding economy. Remedies adopted : establishment of higher schools of education, and of correspondence further training courses for primary teachers.

PORTUGAL. Shortage, due to appeal of technical professions and military careers, and severity of entrance examination to teacher training.

ROUMANIA. No shortage.

SALVADOR. Shortage. Remedies adopted : establishment of a higher school of teacher training with scholarships covering free tuition, board and lodging, and an allowance.

SPAIN. No shortage.

SWEDEN. Acute shortage of secondary teachers of mathematics and natural science. Remedies adopted : further training facilities for primary teachers, creation of a new category of teacher coming between primary teachers and full secondary teachers, short courses in mathematics.

SWITZERLAND. *Geneva.* No shortage. — *Ticino.* No shortage.

SYRIA. Shortage of teachers at all levels of education, due to considerable increase in the number of schools. Remedies adopted : foreign teachers appointed, payment for supplementary hours, the promotion of primary teachers.

THAILAND. No shortage.

UKRAINE. No shortage, teacher training being organized on the basis of estimated future needs.

UNION OF SOUTH AFRICA. Shortage, due to appeal of industry and commerce. No remedies adopted as yet.

U.S.S.R. No shortage, teacher training being organized on the basis of estimated future needs.

UNITED KINGDOM. *England and Wales.* Shortage, due to better paid posts in industry. Remedies adopted : increase of salaries of teachers responsible for advanced courses. — *Northern Ireland.* The position is improving in grammar schools, and is satisfactory in secondary intermediate schools, but there is a shortage in regard to technical intermediate schools ; special committee appointed to find remedies. — *Scotland.* Shortage of both mathematics and science teachers, due to raising of school age, expansion of science teaching, and increase of demand for mathematicians in industry, etc. Remedies envisaged : recall of retired teachers, special scholarships, exemption from military service, better facilities for teacher-trainees, improved conditions of service.

UNITED STATES. Shortage, due to inadequacy of teachers' salaries, lack of prestige of teaching, and length of period of teacher training. Measures adopted : campaigns to increase teachers' salaries, and to raise the prestige of teaching.

VIETNAM. Shortage, due to effects of war, and inadequacy of teachers' salaries. Remedies envisaged : scholarships, increase of teachers' salaries.

YUGOSLAVIA. Shortage, due to increase in secondary school enrolments. Remedies adopted : higher educational institutes and university faculties opened ; scholarships for prospective mathematics teachers.

AFGHANISTAN

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the six years of secondary education. The age range of the pupils is from 11 to 18 years. Five hours a week (one seventh of the total time-table) is allocated to the subject in the 1st to the 5th year inclusive, and ten hours (two sevenths) in the mathematics class and three hours in the philosophy class in the 6th year.

Mathematics appears in the secondary curriculum as arithmetic, algebra, geometry, and, in the upper years, as trigonometry, descriptive geometry, and mechanics also. In examinations the coefficient allotted to mathematics is 2 or, in the mathematics class, 3.

Aims of Mathematics Teaching

The aim of mathematics teaching is to cultivate the scientific attitude, through training the pupils to think logically and clearly and giving them an interest in research.

Syllabuses

1st year. Elements of mathematical reasoning. Arithmetic : metric system, problems on the four rules, the rule of three, percentages, non-decimal units (time, English and Indian money), constant practice in mental arithmetic. Geometry : lines, angles, triangles, calculations of simple surfaces and volumes. — *2nd year.* Revision of work done in the previous year. Arithmetic : prime numbers, decomposition of numbers into their prime factors, practical problems in proportions, simple interest and discount, problems in proportional division, squares and square roots, cubes ; calculation of areas, volumes, densities and weights. Algebra : fundamental notions, use of letters, simple equations of the first degree with one unknown and simple problems thereon, products and their graphical representation. Geometry : properties of parallels, sum of the angles of a triangle, parallelograms, circles, simple constructions. — *3rd year.* Revision of work done in the two previous years. Arithmetic : direct and inverse proportion, ratios. Algebra : positive and negative numbers, and operations on them ; monomials and polynomials ; algebraic fractions and operations on them ; solution of equations of the first degree with numerical coefficients, and application to simple problems ; graphs. Geometry : ratio of two segments, Thale's theorem and applications, similar triangles, properties of right-angled triangles, theorem of Pythagoras

regular polygons and their inscription in a circle ; definition of sine, cosine and tangent of an acute angle in a right-angled triangle ; areas of polygons ; practical application throughout. — *4th year*. The pupils have become familiar with the fundamental notions of mathematics in the lower years, and may now solve problems and discover certain properties for themselves ; their work should no longer be limited to the solution of numerical problems. Algebra : addition and multiplication of polynomials, functions and graphs ; equations and inequations of the first degree with two unknowns, their discussion, and their graphical representation ; problems on simultaneous equations, solution of quadratic equations by completing the square with numerical coefficients only. Geometry : revision ; symmetry and translation ; homology, harmonic division, loci, sine, cosine and tangent of angles from 0 to 180, formulas. — *5th year*. Solid geometry : the theory of and numerical exercises on planes and straight lines (intersections, orthogonality, symmetry, translation, projection) ; volumes and areas (prisms and pyramids, spheres, cylinders, cones, section of a sphere) ; tangents and tangent planes. Algebra : quadratic equations, existence and signs of the roots, inequations, finding two numbers given their sum and products, application of graphs to quadratic functions. Trigonometry : circular functions of an arc, functions of supplementary and complementary angles ; sine, cosine, tangent ; geometric sum of vectors, and their projection on an axis ; formulas for addition of angles and multiple angles. — *6th year (mathematics class)*. Geometry : vectors, projections, geometric sum, scalar products, harmonic division, pole and polar, symmetry, translation and rotation, homology, similitude, inversion, ellipse, hyperbola and parabola, section by a line, tangent, problems and sections of a cone. Descriptive geometry : point, line, plane, intersections, parallelism, representation of prisms and pyramids, plane sections, lines and perpendicular planes, change of planes ; application to problems of distance and angles. Algebra : revision ; operations on polynomials (including division), radicals, irrational expressions, equations and inequations of systems of the first degree or reducible to the first degree, equations and inequations of the second degree ; functions, graphs ; variation, derivatives, tangents to a curve, quotients of binomials and trinomials, integrals of simple polynomials and their application to areas. Trigonometry : circular functions, addition and multiplication formulae, simple trigonometrical equations ; solution of triangles and use of tables. Mechanics : kinematics : rectilineal motion, diagrams of velocities and accelerations, curvilinear motion, velocity and acceleration, vectors, application to a circular uniform motion, simple harmonic motion ; statics : equilibrium on a plane, on a line, on a circle, and on a sphere ; reaction, friction. — *6th year (experimental science class)*. Algebra : equations, inequations and systems for the first degree ; revision of trinomials ; homographic functions of the second degree, functions, derivatives, tangents to a graph, simple applications ; integrals related to the area bounded by a curve, two ordinates and the x-axis ; definition of the log and of the exponential by the integrals. Trigonometry : circular functions, additions of angles and multiple angles, trigonometrical calculations, numerical solution of triangles. Mechanics : rectilineal motion, curvilinear motion, velocity and acceleration, vectors, simple harmonic motion. — *6th year (literary class)*. Exercises on the syllabus in algebra for the 4th and the 5th years ; circular

functions and periodicity, derivatives, tangent to a curve, application of the sign of a derivative to the variations of simple functions.

The syllabuses do not cover the history of mathematic or the principles of axioms.

Methods of Teaching Mathematics

The methods employed for the teaching of mathematics are those used in French secondary schools. The teachers are given instruction in them during their teacher training. The teachers of mathematics, moreover, are responsible for the correlation of their subject with other subjects in the secondary curriculum.

Neither films nor filmstrips are used for the teaching of mathematics. The material necessary for practical exercises (surveying and simple constructions) is provided. The use of a textbook is compulsory.

Teachers

All secondary teachers are required to have completed not less than the baccalaureate course ; many of them hold a first degree or a doctorate.

No difficulty is experienced in recruiting secondary teachers of mathematics. Such teachers obtain further training through reading the educational journals and through the grant of scholarships for further study.

AUSTRALIA

From the reply sent by the Commonwealth Office of Education [1]

Place of Mathematics in Curriculum

Mathematics is taught in each year of the secondary course, which varies in length as between the states. The full course is five years in length in New South Wales, South Australia, Western Australia and Tasmania, six years in Victoria, and four years in Queensland. In all the states, moreover, it is possible to take a course of shorter length.

Pupils are about 13 years old (a year less in Victoria, a year more in Queensland) at the beginning of the secondary course, and about 17 years old in the 5th year.

Of the total twenty-four or twenty-five hours spent in class each week, the time allocated to mathematics is between four and six hours (i.e., between one sixth and one quarter of the total time) in classes taking mathematics as a general subject in one unit, between six and eight hours (i.e., between one quarter and one third of the total time) in classes taking mathematics in two separate units, and about ten hours (i.e., about two fifths of the total time) in those cases where it is possible in the later years of the course to take mathematics as three separate units.

Mathematics is taught under the following headings:— *1st year:* arithmetic, algebra, and geometry. — *2nd year:* arithmetic, algebra, geometry, and (Queensland only) trigonometry. — *3rd year:* arithmetic (Queensland excepted), algebra, geometry, and trigonometry. — *4th and later years:* algebra, geometry, trigonometry, differential calculus, plane analytic geometry and algebraic geometry, and (as an extra subject unit) mechanics.

Mathematics instruction is compulsory for the first two or three years of the secondary course, but need not be taken in the 4th and later years of the course. The subject organization of a school in many cases is such that the option to take mathematics depends on the choice of other subjects.

There are, moreover, alternative courses in all states in which mathematics is taught at a more elementary level, on the one hand, and more intensive study of the subject in the 4th and later years is available for pupils specialising in mathematics and attempting honours passes in final examinations, on the other.

In both promotion examinations and the secondary school leaving examinations, mathematics commonly may count as either two papers or one, depending on the pupil's course of study, while with few exceptions other subjects count as one (although two languages or sciences may be taken, of course, each counting as one unit).

[1] The reply covers only government schools, but the non-goverment schools generally follow their practice.

Aims of Mathematics Teaching

The aims assigned to the teaching of mathematics may be found in the syllabuses and suggestions to teachers. The New South Wales " Syllabus in Mathematics ", for example, states that the courses should aim at helping pupils " to interpret their environment from a quantitative point of view " and to acquire some knowledge and skill in dealing with " the quantitative aspects of those social and economic questions that are the common experience of an ordinary enlightened citizen ", and it makes reference to the interest that mathematical knowledge may arouse on account of " its usefulness in a vocation ".

Syllabuses

1st, 2nd and 3rd years. Arithmetic: the four fundamental operations, highest common factor and lowest common multiple, vulgar and decimal fractions, percentages, ratio and proportion, weights and measures; applications to money, interest, discount, etc., profit and loss, graphs. Algebra: the use of symbols, simplification with brackets, multiplication and division, identities, simple equations, fractions, use of formulas, graphs (including their use in the solution of simple quadratic equations), solution of quadratic equations, solution of two or three simultaneous equations, graphs (including maxima and minima and the solution of simultaneous equations). Geometry: propositions, definitions and axioms (relating to angles and parallels, angle sums of triangles and polygons, theorem of Pythagoras, areas of various kinds); constructions with ruler and compass (including bisection of angles, straight lines and arcs of a circle, circles, including circles inscribed and escribed in a triangle, tangents to a circle). Trigonometry: introduction to the ratios sine, cosine and tangent; use of tables; problems involving one right-angled triangle. — *4th and later years.* Algebra: rational and irrational numbers, general theory of quadratic equations, indices, ratio and proportion, arithmetical and geometrical progression, graphical work, identities, interest including present value of annuities, permutations and combinations, etc., differential and integral calculus and applications (to equations of tangents of curves, maxima and minima, rates of change, areas, volumes and solids of revolution). Geometry (including analytical geometry): ratio and proportion, similarity in triangles and polygons; theorems, problems and constructions on bisections of the sides and angles of a triangle, circles, tangents and chords; equations of the conics (not in all states). Trigonometry: the six trigonometrical ratios and their relationships, solution of triangles using logarithms, solution of trigonometrical equations, graphical representation of trigonometrical functions, trigonometrical expressions; problems on all the foregoing. Mechanics: statics and dynamics in two-dimensional situations.

The history of mathematics and the principles of axioms may be briefly touched on during the lessons, but are not included in the syllabuses.

Methods of Teaching Mathematics

No official instructions are issued concerning the methods of teaching mathematics in secondary schools. As already noted, however, the syllabuses generally contain suggestions to teachers in this connection ; the New South Wales mathematics syllabuses, for example, recommend teachers to appeal to the pupils' interests, and to adapt the difficulty of the work to individual capacity. Advice is given, moreover, by the inspectors of the Education Departments, and all states have facilities for the training of teachers which includes methods of teaching mathematics.

The most common method of teaching is that of instruction and demonstration by the teacher, followed by individual work on exercises set and corrected by the teacher. Less frequently, project and study assignment methods are used by individual schools or teachers.

There are no special instructions bearing on methods of teaching mathematics at the transfer stage from primary to secondary education. The methods followed at this stage are the same as at other stages, but revision of primary school work is undertaken along with new work in the 1st secondary year.

The mathematics syllabuses are planned as part of a planned curriculum, and as such are coordinated with other subjects. Teachers may at their own discretion plan their teaching to fit in with the work of teachers of other subjects.

Two Education Departments produce filmstrips for use in the teaching of mathematics, and teaching charts. No motion films on the teaching of mathematics have been made in the country, but some from abroad are available from the central office of various Departments.

Mathematics is shown in many applications at all stages of instruction. Arithmetic, for example, is applied to monetary and measurement problems, trigonometry to problems in land measurement, and so on. Applied mathematics in this sense is thus an integral part of the course. The term " applied mathematics ", however, is generally reserved for courses in mechanics, which is optional in all states and is not usually introduced before the 4th year.

Practical exercises do not generally precede, but rather accompany and exemplify, formal teaching. At his discretion the individual teacher may conduct practical exercises beforehand, and use activity or project methods, but the practice is not common.

Lists of textbooks suitable for pupils and for teachers are printed in the various school syllabuses, but the choice of textbooks is left to the individual school.

Teachers

In general the mathematics teacher must have basic training (including supervised practice) as a teacher, special training in the methods of teaching mathematics, and a university degree with mathematics taken as part of the course. In some cases students who have not completed their university course teach mathematics in secondary schools (usually in the lower classes only), but they must have had training as teachers.

There is a general shortage of teachers, and this shortage is possibly more acute in regard to mathematics and science than in regard to other subjects. This is problably due, in part at least, to the attractiveness of other avenues of employment for people with a mathematical and scientific training. In an effort to overcome the general shortage of teachers, there have been recruiting campaigns and special programmes of financial assistance for prospective teachers at the secondary level. Financial assistance for teacher trainees doing university and professional training is a regular feature of teacher training programmes in all states.

Exchange of teachers with other countries and between the states of the Commonwealth of Australia does take place, but the number of mathematics teachers involved is very small. Refresher courses and seminars are sometimes held for mathematics teachers who care to attend.

The " Mathematical Association " which exists to further the study and teaching of mathematics publishes the periodical, " The Australian Mathematical Teacher ", which is largely devoted to notes and articles dealing with teaching techniques and the treatment of topics in the secondary school syllabuses. One State Department of Education publishes a mathematical bulletin for teachers in secondary schools, the contributions coming mainly from the teachers themselves.

Miscellaneous

The methods of teaching mathematics are modified from time to time, but as far as is known no sweeping changes are being planned at the moment.

All secondary mathematics syllabuses have been affected by modern developments in the mathematical field to the extent that they include, for example, calculus and analytical or algebraic geometry. It would, however, be true to say that the main changes that have been made in recent years in them have resulted from advances in educational thinking rather than from those developments.

Most of the experiments being carried out at the moment are being undertaken by undergraduate and graduate students at universities. They are concerned with a variety of topics including the construction of diagnostic tests, studies of the causes of failure in particular areas of mathematics, the mathematical needs of pupils, and the attitudes of pupils towards mathematics.

BELGIUM

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Secondary education follows six years of primary education, and beginning at the age of twelve, covers a period of six years, divided into two three-year cycles.

The lower cycle has two sections : (a) a modern section ; (b) a classical section. In the upper cycle the classical section is divided into Latin-Greek, Latin-science, and Latin-mathematics sub-sections, and the modern section into modern-scientific and modern-commercial sub-sections. In lower secondary schools there are also (a) domestic sections (for girls), (b) pre-agricultural and pre-vocational sections.

All sections of the lower cycle have the same programme in mathematics. Four hours a week is allocated to the subject, which constitutes 11.1% of the total time-table in the Latin section and 12.5% in the modern section.

In the upper cycle three hours a week is allotted to mathematics in the Latin-Greek sub-section, seven hours in the Latin-mathematics sub-section, five hours in the Latin-science sub-section, seven hours in the modern-scientific sub-section, and three hours in the modern-commercial sub-section.

Mathematics embraces arithmetic, algebra, and geometry (intuitive geometry only in the 1st year) in the lower cycle, and arithmetic, algebra, geometry (together with analytical and descriptive geometry in the last two years of the modern-scientific and the Latin-mathematics sub-sections), and trigonometry, in the upper cycle.

Mathematics is a required subject in all years. The same importance is attached to it as to other major subjects in promotion and school leaving examinations.

Aims of Mathematics Teaching

The principal aims of mathematics teaching at secondary level are to cultivate the pupils' capacities for logical thinking, analysis, and observation, and their creative imagination, to train them scientifically, and to enable them to consider methodically and objectively all mathematical questions arising in the course of their professional work or their everyday life. The acquisition of good ways of working is considered of much greater importance than the burdening of the memory with a great store of knowledge.

Syllabuses

Lower Cycle :

A. *General Education Sections. 1st year.* Arithmetic : the fundamental operations with whole numbers and fractions, highest common factor and lowest common multiple. Geometry : an intuitive study of the

main properties of common plane and solid figures. — *2nd year.* Arithmetic : the operations with whole numbers and fractions ; problems involving mixtures, alloys, interest, and discount ; the use of diagrams. Algebra : relative numbers, algebraical calculations up to the division of a polynomial by a monomial. Geometry : introduction. — *3rd year.* Arithmetic : ratios and proportions, relations between the highest common factor and the lowest common multiple of two whole numbers, square root. Algebra : division of a polynomial by a polynomial, factorization, the operations with rational fractions, first-degree equations, graphs. Geometry : the circumference, proportional lines, similarity of triangles and polygons, the measurement of areas, the dimensional relationships of the elements of a right-angled triangle, the trigonometrical relationships of an acute angle.

B. *Domestic Section. 1st year.* Arithmetic : the four operations, revision of the metric system, simple problems. Geometry : the basic properties of common plane figures. — *2nd year.* Arithmetic : revision of the operations with fractions, identities, ratios and proportions, square root. Geometry : the basic properties of common solids. — *3rd year.* Arithmetic : the operations with non-decimal quantities, the use of diagrams. Algebra : simple equations of the first degree. Geometry : an introduction to the straight line, the segment, the angle, etc. ; the construction of perpendiculars, bisectors, medians, the congruence of triangles, introduction to the similarity of triangles.

C. *Pre-agricultural and Pre-vocational Sections.* The same syllabuses as for the general education sections, but with more practical exercises to meet the demands of elementary technics.

Upper Cycle :

A. *Latin-Mathematics and Modern-Scientific Sub-Sections. — 1st year.* Arithmetic : the properties of the multiplication and division of whole numbers, square root. Algebra : the principles of the equivalence of equations, inequalities, and systems of equations ; the algebraical and the graphical solution of equations of the first degree with one unknown, arithmetical and geometrical progressions, introduction to logarithms. Geometry : revision of the properties of plane figures, loci and constructions, regular polygons, the circumference, the area of a circle and the circular sector, the theorems of Menelaus and Ceva. Trigonometry : the definition and graphical representation of trigonometrical functions, fundamental formulas, formulas of the addition and multiplication of arcs, the solution of trigonometrical equations and triangles. — *2nd year.* Arithmetic : the conversion of vulgar fractions into decimal fractions, introduction to the theory of errors. Algebra : equivalent and identical polynomials, the method of indeterminate coefficients, combinatory analysis (simple groups), the binomial theorem of Newton, determinants, limits, continuity, the derivate of a function $y = f(x)$, application to the variations of the function of a variable. Geometry : areas and volumes, the intersection of straight lines and planes ; poles and polars in the circle, the theorem of Pascal and Brianchon, an introduction to analytical geometry (the system of coordinates, an introductory study of the straight line and the circle). Trigonometry : the solution of angles in uncommon

cases. — *3rd year.* Arithmetic : the theory of divisibility, the prime numbers, the theorem of the highest common factor and the lowest common multiple, the number of the factors of a number. Algebra : an introduction to irrational numbers ; complex numbers, the number " e ", exponential and trigonometrical functions. Geometry : the transformation of figures by homology and inversion, analytical geometry (the general theorem of curves of the second degree, tangents, polars, asymptotes, centres and diameters, the axes of symmetry, foci, conical pencils, special properties of the three types of conic sections, polar coordinates).

B. *Latin-Greek and Modern-Commercial Sub-Sections.* The syllabuses are not so wide as those of the Latin-mathematics and modern-scientific sections, and do not include arithmetic in the 2nd or the 3rd year, or any analytical or descriptive geometry. In algebra, the work on arithmetical and geometrical progressions and logarithms is given in the 2nd instead of the 3rd year, and that on limits and the derivate of a function $y = f(x)$ in the 3rd instead of the 2nd year. The work in geometry ends with the areas and volumes of polyhedrons and round bodies. In trigonometry, the work on the addition and multiplication of arcs is given in the 3rd instead of the 1st year. An optional course, moreover, in commercial algebra is offered in the 3rd year of the modern-commercial section (compound interest, annuities, depreciation).

C. *Latin-Science Sub-Section.* The syllabuses for the 1st and the 2nd year are very similar to the syllabuses for the corresponding years of the Latin-mathematics section. There is no arithmetic, however, in the 2nd year or descriptive geometry in the 3rd year ; the 3rd year algebra syllabus, moreover, includes elements of differential and integral calculus (the differential of a function, the theorems of Taylor and Mac-Laurin, the finite and infinite integrals, the principal methods of integration, the calculation of areas and volumes).

The syllabuses do not include the history of mathematics or an introduction to the principles of axioms. Teachers generally make some general references, however, to the principles of axioms in the upper years of the various sections.

Methods of Teaching Mathematics

A committee was appointed in 1947 to adapt the mathematics syllabuses and the methods of teaching the subject to the mental ages of the pupils, and to propose means suitable for ensuring the pupils' active intellectual participation. This committee recommended that teachers should make use whenever necessary of concrete examples, on the grounds that abstract ideas are usually more easily understood if they are taught on a simple and natural intuitive foundation. In the 1st year, for example, geometry should be introduced in an essentially intuitive and practical way. Then, in and after the 2nd year, intuition should be gradually replaced by geometrical reasoning, and the work done in the preceding years should be gone over again and presented in logical sequence and with a view to the classification and synthesis of the concepts involved.

The same committee also drew teachers' attention to the value of studying loci and of constructional problems in developing the pupils'

power of imagination and ability to do research. Most teachers, acting on the principle that intuition and sense experience often form natural channels towards discovery, get their pupils to cooperate in making things illustrating the work in geometry. Through the remarkable development of science, moreover, the concept of function is of ever-increasing importance, and plays a leading part as a result in the teaching of algebra, in connection with the laws of physics.

Endeavours are being made to replace the traditional methods of teaching mathematics with more modern methods, by a group of mathematicians associated with the journal *Mathematica et Paedagogica*. The methods they favour are based on modern psychology and modern trends in the mathematical field.

The Department of Education is actively interested in the problem of the vertical correlation of subjects and teaching methods in the period of transition from primary to secondary education. The teachers revise the work done at primary level, classify it, take it a step further, and gradually lead up to the methods and the spirit of secondary education; they make use of as great a variety of approaches in their teaching as possible, and frequently employ activity methods, which best stimulate industry and individual initiative.

Secondary mathematics teachers have frequent opportunities of correlating their subject with other subjects in the curriculum, physics, in particular, in the upper years. Quite often one and the same teacher is responsible for certain aspects of both mathematics and physics.

Some teachers of the lower years make satisfactory use, in their work on geometry, of figures for the study of loci and to show the number of elements required for the determination of a figure. Some Swiss and French films are available for illustrating the theorems of the functions of the first and the second degree.

Emphasis is laid on intuitive ways of learning. In schools where such methods are employed, the teaching materials are generally made by the pupils themselves, and not simply presented by the teacher.

The teaching of geometry in the 1st year is based on work done by the pupils in handicrafts lessons. The pupils make full-scale or reduced models of plane and solid figures. There is a strong movement in favour of using models for the teaching of geometry in the succeeding years as well. The materials needed by the mathematics teacher for studying the properties of solids and figures, are prepared by the handicrafts teacher.

The concrete and practical aspects of the mathematics course are naturally given even greater emphasis in the pre-technical sections.

All textbooks for use in State schools are subject to the approval of the *Conseil de perfectionnement*, in the light of this committee's evaluation of their scientific and educational worth. Teachers are not allowed to abandon the textbook and dictate long notes to their pupils, which would destroy individuality and make the pupils merely passive recipients.

Teachers

A mathematics teacher in a lower secondary school is required to hold the *agrégé* certificate of lower secondary teaching (science section)

awarded by the secondary teacher-training schools. These schools provide thorough professional training, and admit (a) young men and women who have completed their secondary education, and (b) men and women primary teachers. The course is a two-year one.

Mathematics teachers in the *athénées* and the *lycées* (offering both the lower and the upper cycle of general secondary education, for boys and girls respectively), are required to hold a first degree in mathematics, and the *agrégé* certificate of upper secondary teaching. The certificate is awarded by all of the four Belgian universities on completion of a four-year course following university studies in science or Latin-mathematics. The professional training embraces both theory and practice. During the first two years of the course students are required to give lessons in a State school, under the supervision of a professor of the methodology of mathematics, and of experienced teachers. The final examination covers the methodology of mathematics teaching, and two lessons on subjects in the upper secondary curriculum.

Considerable difficulty is experienced in recruiting secondary teachers of mathematics. Most of the students graduating in mathematics, a subject reputed to be very difficult, are drawn into engineering and other professions which offer them numerous openings in Belgium and in the Belgian Congo and other countries abroad, and are more remunerative than teaching. It nevertheless appears that the number of students now enrolled in science-mathematics courses offers hope for the future.

The opportunities for further training available to secondary teachers of mathematics include the following : (a) the visits of the inspectors, and teachers' meetings presided over by an inspector, at which the problems of correlation, methods and organization arising out of the inspection are discussed ; (b) the meetings and conferences organized by the Belgian association of secondary mathematics teachers, which was founded two years ago and now has 500 members, and the journal published by this association, which has given satisfaction both to Belgian teachers and to teachers abroad.

The third conference organized by the same association will be held in November, 1956, on the subject of the concrete and intuitive sources of the principles of mathematics. On the occasion of the conference the Ministry of Education will organize an exhibition of models.

Miscellaneous

No modifications are at present envisaged in the teaching of mathematics in the general education sections ; insufficient time has as yet elapsed for a complete evaluation of the results of the reform of syllabuses and methods begun in 1946. Considerable modifications, however, are being made in the syllabuses and time-tables of the pre-technical sections, with a view to their coordination with the syllabuses and time-tables of technical education.

A plan to increase considerably the scientific training given to lower secondary teachers is under consideration.

BULGARIA

From the reply sent by the Ministry of Foreign Affairs

Place of Mathematics in Curriculum

Mathematics is a compulsory subject throughout the four years of secondary education (8th-11th years of schooling inclusive, pupils 14-17 years of age). Five hours a week (16.67% of the full time-table) is devoted to it in the 1st, the 2nd, and the 3rd year, and four hours a week (13.33% of the full time-table) in the 4th year.

Mathematics appears as algebra and plane geometry in the 1st and the 2nd year, as algebra, stereometry and trigonometry in the 3rd year, and as algebra and trigonometry in the 4th year.

Mathematics is regarded as being a basic subject, and special importance is attached to it in consequence, as compared with other subjects. It is included in the examinations coming at the end of the 7th year of schooling (the point of transition from primary to secondary education), and in those coming at the end of secondary education.

Aims of Mathematics Teaching

The aims assigned to mathematics at secondary level are to give pupils knowledge which will prove of definite use to them in everyday life, and, when necessary, to prepare them for higher studies.

Syllabuses

Pupils begin a systematic study of algebra and geometry in the 7th year of primary education. The main points in the mathematics syllabus at secondary level are as follows. *1st year.* Simple equations, linear functions, systems of linear equations. Geometry : rectangles, circumference. — *2nd year.* Approximations, square root, series of numbers, second-degree and third-degree equations, irrational equations. Geometry : similar figures, regular polygons, areas. — *3rd year.* Systems of second-degree equations in two unknowns, progressions, concept of the degree of equations, exponential function, logarithms, compound interest. Geometry : introduction to geometry in space, position of lines and planes in space, round bodies, polyhedrons, trigonometrical functions of an angle. — *4th year.* Complex numbers, inequalities, revision of the algebra syllabus. Trignometry : solution of triangles.

The secondary mathematics syllabuses make no reference to the history of mathematics or to the principles of axioms.

Methods of Teaching Mathematics

Teachers of mathematics are recommended in official suggestions to make use of introductory talks and exposition, in addition to purely formal teaching. The latter assumes most importance in the final year.

With a view to facilitating the transition from primary to secondary education, the secondary teachers have made recommendations to primary teachers concerning methods of teaching mathematics. The algebra syllabus, moreover, of the final primary year (the 7th) is revised at the beginning of secondary studies. Further, a constant endeavour is made to relate the teaching of mathematics to that of other subjects, more especially in connection with chemistry and physics problems.

The materials used for mathematics lessons include drawing boards, diagrams, and models, but not films or filmstrips. The lessons include practical exercises, above all in the form of surveying in connection with geometry and trigonometry.

Teachers are required to use the textbooks approved by the Ministry of Education.

Teachers

Mathematics is taught at secondary level by teachers who have completed the full course of the faculty of mathematical and physical sciences of the State university (mathematics division), and have passed the official examination. No difficulty is experienced in recruiting teachers.

Mathematics teachers can secure further training, both academic and pedagogic, through the courses, seminars and workshops organized at the institute specially established for providing further training, and through the summer courses, periodical conferences, and meetings arranged by the teachers' associations.

Miscellaneous

The information given above derives from recent study plans (approved by the Council of Ministers on 15th March, 1955), and no changes are therefore to be expected in the near future. These study plans take account of the latest developments in mathematics.

Mathematics is taught in the light of the requirements of polytechnical education, and this encourages the abundant use of experiments, the best of which are given quite special attention.

A book on methods of teaching mathematics, by P. Ivanov, was published in 1954, and a collection of articles on the same subject is now in the press.

BYELORUSSIA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

The last six years of the course of the ten-year schools constitutes secondary education. The age range of the pupils is from 12 to 17 years. Six hours a week (about 20% of the total time-table) is allotted to mathematics in each year.

The subject embraces arithmetic in the 1st year (the fifth year of schooling), arithmetic, algebra and geometry in the 2nd year, algebra and geometry in the 3rd and the 4th year, and algebra, geometry and trigonometry in the 5th and the 6th year.

The secondary curriculum does not include any optional subjects, but pupils may take part in the work of the school societies, which is done outside class hours, and further details of which are given below in the " Miscellaneous " section.

Mathematics ranks as a major subject on account of the part it plays in general education and of its practical applications, and the pupils are examined in mathematics at the end of every school year.

Pupils seeking admission to physics-mathematics or technical courses at higher level are required to pass an entrance examination in mathematics.

Aims of Mathematics Teaching

The teaching of mathematics satisfies the demands of both general and polytechnical education.

The subject, moreover, is a good preparation for all the sciences, and mathematical knowledge is a pre-requisite for the study of physics, astronomy, chemistry, geography, economics, etc. Mathematics teaches the pupils to act and to reach conclusions on the basis of concrete facts ; it accustoms them to accuracy in speech, orderliness, concentration, perseverance, and critical thinking. A widespread knowledge of mathematics leads to a more conscious and efficient participation by each person in the realization of socialism. The study of mathematics involves work on judgment-forming abstract problems, and on practical problems over a wide field, and is of fundamental importance in Soviet schools.

Syllabuses

The arithmetic course in the 1st year (the 5th year of schooling) embraces the following : revision of the work done at primary level (the four operations, the metric system) ; vulgar and decimal fractions, highest common factor and lowest common multiple, elementary geometry ; practical work in the field. To these materials are added the rule of three and interest, in the 2nd year.

The algebra course embraces the following : the algebraical expressions, equations of the first degree with one unknown, positive and negative numbers, the operations (as far as polynomials and multipliers) ; functional dependence and its graphical representation ; applications to simple problems of physics, chemistry, astronomy, agriculture, etc. In the 3rd year, algebra is taught in the following order : development of polynomials and multipliers, algebraical fractions, proportion and proportional dependence, equations of the first degree with one unknown, systems of equations of the first degree, the construction of graphs. The 4th year syllabus covers the following materials : powers and roots, irrational numbers, second-degree equations, quadratic trinomials, systems of equations of the second degree with two unknowns. The 5th year syllabus embraces the limits, progressions, finite and logarithmic functions, and demonstrative and logarithmic equations. The 6th year syllabus covers the binomial theorem of Newton, complex numbers and the operations with inequalities of the first and the second degree, inequalities, etc.

The geometry course includes instruction in the general theoretical and practical significance of geometry. The materials it covers are as follows : 2nd year (the 6th year of schooling) : the basic elements (figures, straight lines, loci) ; 3rd year : the properties of parallelograms and trapezia, the dimensional relationships of the elements of the polygon and the circle, the measurement of angles, the properties of inscribed and circumscribed triangles and quadrilaterals ; 4th year : the ratios and proportions of segments, similarity, the trigonometrical functions of the acute angle, the dimensional relationships of the elements of the triangle, the area of rectilinear figures ; 5th year : theorems on inscribed and circumscribed regular polygons, the length of the circumference and the area of the circle, solid geometry (as far as polyhedrons, and problems of construction in space) ; 6th year : polyhedrons, solids of revolution, the area and volume of solids of revolution.

The trigonometry course covers the following : 5th year : basic concepts, and the formulas of dependence among the trigonometrical functions of any angle, the measurement of arcs and angles, the formulas of reduction and transformation, logarithmic tables of trigonometrical functions ; 6th year : obtuse angles, inverse functions, trigonometrical equations, the solution of geometrical problems with the aid of trigonometry.

The pupils are given an idea of the history of mathematics, and of the life of some famous mathematicians of the U.S.S.R. and countries abroad. They are also given an introduction to the principles of axioms.

Methods of Teaching Mathematics

Secondary teachers of mathematics carry out the recommendations concerning methods of teaching the subject, which are given in the mathematics syllabuses and in certain official publications on arithmetic, algebra, geometry, and trigonometry. The recommended methods conform to the general principles of Soviet pedagogy, i.e., they help to promote a scientific attitude in the pupils, and to lead them towards a global concep-

tion of phenomena (generalities on non-Euclidean geometry, etc.). Mathematics is presented as a first-class instrument for the investigation of the material world leading both to the study of natural phenomena and the discovery of means of controlling them.

No single method is regarded as being universally applicable. The most commonly used method is systematic presentation of the material by the teacher ; the exact procedure varies according to the problem, and the age and training of the pupils. Frequent use of directed discussion is made in the lower years. On other occasions the analysis of problems is made by the class, the teacher intervening to draw the argument together ; variations of this method are to be found, according to the degree to which the teacher intervenes. The method of lecture-lessons is used to some slight extent in the upper years.

Chief emphasis is placed on pupils being able to solve their problems, to use their textbooks, and to organize their homework.

The Byelorussian Ministry of Education has issued instructions concerning the period of transition from primary to secondary education. Due account of the special needs of this period is taken in the mathematics syllabuses, textbooks, and guides on methods, especially as regards the 1st year (the 5th year of schooling), when the pupils are taught mathematics for the first time by a specialist teacher.

The syllabuses drawn up and approved by the Ministry of Education ensure the correlation of mathematics with other subjects. Due account is taken of the interdependence of the various subjects in the syllabuses for each year of the secondary course. Teachers are given instructions in this connection.

The teaching materials and auxiliary aids for mathematics lessons include geometrical figures and solids, models illustrating problems or proofs of theorems, charts, instruments for practical work, calculating machines, and films.

The practical work done comprises surveying, geodesy, plan drawing, the measurement and calculation of volumes and areas, the making of models, and the drawing of graphs, charts, diagrams, etc.

The syllabuses lay down that the pupils shall become familiar with geometrical forms through the study of natural or artificial models, and learn to use the simpler instruments of measurement.

An adequate supply of textbooks for both pupils and teachers is published by the Ministry of Education.

Teachers

Secondary teachers of mathematics are required to have completed the physics-mathematics course of a teacher training school or a university. Such a course lasts four or five years, and includes the history and the theory of education, the methodology of mathematics and allied subjects, and periods of teaching practice under competent supervision, towards the end of the course.

There is a sufficient number of candidates for secondary mathematics posts, as the choice of higher studies is determined by the educational development plans.

Three-day meetings at district level are organized three times a year, at which secondary mathematics teachers discuss their professional problems. Education committees, moreover, are to be found in each secondary school, for the study of the methods to be used in the teaching of each subject ; the teachers thus have an opportunity of exchanging viewpoints, and they participate in lessons given by their colleagues. Further training courses and seminars are organized periodically.

Courses, conferences, and practical work are organized at various administrative levels by the Ministry of Education, the regional further-training institutes, and the district pedagogical centres. At the annually held " pedagogical readings ", secondary mathematics teachers have opportunities of submitting reports, some of which are subsequently published.

Secondary mathematics teachers read such journals as " School Mathematics ", " The Teacher's Guide ", and " Soviet School ", and can publish articles in them. The Ministry of Education makes teachers an allowance for the purposes of attending courses and conferences.

Miscellaneous

A revision of the secondary mathematics syllabuses has been in progress since 1954-1955. It involves a better distribution of the various topics in relation to their importance and the age of the pupils, the replacement of certain topics by more important ones, greater attention to the correlation of mathematics with other subjects and to practical work, and the revision of some of the textbooks.

Modern developments in the mathematical field have not directly affected the secondary mathematics syllabuses, but stress on functional dependence throughout the course is considered to be an important factor in general education and the development of logical thought. The same is true of certain aspects of analytical geometry, which assist the pupils to assimilate some of the concepts of higher mathematics.

The attempts being made to raise the standard of mathematics teaching mainly involve the improvement of teaching methods and textbooks, and increased correlation between the teaching of mathematics and polytechnical education. Some teachers are experimenting with an advanced study in their classes of various general and particular problems. Reports on the progress made in this way are made in the specialist journals and at educational conferences.

In accordance with a carefully devised plan, a sufficient quantity of handbooks on all aspects of the teaching of mathematics is published by the Ministry of Education. Many Russian publications of the R.S.F.S.R. Academy of Educational Science are re-published in Byelorussian. Each year a list of the Ministry's handbooks is published in the press, and brought to the notice of schools. The revision of the secondary mathematics syllabuses, referred to above, will probably be finished by 1957-1958. The decision to make this revision was taken at the same time as the decision to make compulsory ten-year schooling universal and to introduce polytechnical education.

Pupils with special interest in mathematics can obtain instruction in it outside class hours. Such study is optional, and gives the pupils opportunities of studying certain problems and the history of mathematics and the lives of mathematicians in greater detail. The work is done through study groups, circles for making teaching materials, soirées, etc. The best pupils compete in " mathematical olympiads " for the solution of complicated problems. They are held at various local levels and also at national level. Books on science are specially published for the use of the school societies.

CAMBODIA

From the reply sent by the Secretary of State for National Education

Place of Mathematics in Curriculum

Mathematics is taught throughout secondary education, from the 1st year *(sixième)* to the classes preparing for the second part of the baccalaureate. The average ages of secondary pupils run from 14 to 21 years.

The weekly time allocation for mathematics is as follows : 2 hours in the 1st year (8% of the total time-table) ; 2½ hours in the 2nd year (10%) ; 3 hours in the 3rd and the 4th year (12%) ; 4 hours in the 5th and 6th year (15.4%) ; 7th year (the final year of preparation for the second part of the baccalaureate) : 9 hours in the mathematics class, 4 hours in the experimental science classe, and 1½ hours in the philosophy class.

It will be seen that mathematics is not given great importance in the first four years of the secondary course, priority being given to Khmer and French. In the mathematics class in the final year, on the other hand, mathematics occupies 32% of the total time-table.

Mathematics is a compulsory subject in all years. It has the same importance as French and Khmer in promotion examinations, and as the basic subjects (science, French, Khmer) in the first part of the baccalaureate, and is the most important subject for the mathematics class in the second part of the baccalaureate.

Aims of Mathematics Teaching

The aims of teaching mathematics are to give the pupils a good mathematical education, and the clarity, exactitude and logic necessary for the study of mathematics, and to prepare them for the study of physics, chemistry, etc.

Syllabuses

The mathematics syllabuses now in force are the same as those in France. — *1st year (sixième).* Arithmetic and geometrical drawing : exercises based on measures of length, area, volume, weight, and time. — *2nd year (cinquième).* Arithmetic : study of divisibility ; introduction to the study of algebra. Geometry : straight lines, angles, triangles, cases of congruence. — *3rd year (quatrième).* Arithmetic : resolution of whole numbers into prime factors. Algebra : elements of algebraic calculation leading up to solution of first-degree equations. Geometry : triangles, polygons, circle. — *4th year (troisième).* Arithmetic : ratio, proportion, square roots. Algebra : first-degree equations, graphical representation.

Geometry : loci, similar triangles, polygons, circle, measurement of area. — *5th year (deuxième)*. Revision ; second-degree equations. Geometry : revision and more advanced work than in previous years. — *6th year (première)*. Algebra : second-degree equations, second-degree trinomials. Geometry : straight lines, planes, volumes, projective geometry. Trigonometry : definitions of circular functions, formulas for the solution of trignometrical equations. — *Philosophy class*. Algebra and trigonometry : revision of the previous year's work ; study of derivatives, graphical constructions, functions. Cosmography : diurnal movement, earth, sun, moon. — *Experimental science class*. Cosmography and mechanics : same as for mathematics class. Algebra : derivatives, study of variations of elementary functions. Logarithms : exponential functions. Trigonometry : circular functions, derivatives of circular functions. — *Mathematics class*. Arithmetic : enumeration, prime numbers, fractions, square roots. Algebra : functions, derivatives, logarithms. Trigonometry : revision ; transformation formulas, trignometrical equations. Kinematics : rectilinear movement, circular movement, vectors, velocity, acceleration. Statics : notion of mass, equilibrium of a point. Geometry : transformations, cones. Descriptive geometry : point, straight lines, planes. Cosmography : earth, sun, sun-moon system.

Reference is made to the history of mathematics only in the final year preparing for the second part of the baccalaureate.

Methods of Teaching Mathematics

There are no official suggestions or instructions concerning the methods to be used in the teaching of mathematics.

Integration between primary and secondary education is effected during the first year of the secondary course.

The syllabuses of the physical sciences and mathematics are coordinated, so that on beginning a study of the former the pupils possess the necessary mathematical knowledge.

The teaching of mathematics does not include practical applications.

There are no official suggestions or instructions concerning the choice or use of mathematics textbooks. Those in use are French textbooks.

Teachers

Mathematics is taught by primary teachers in the lower secondary classes, and by French secondary teachers, with a first degree or a teaching diploma, in the upper secondary classes.

No arrangements exist for the further training of mathematics teachers.

Miscellanenous

No changes are at present envisaged in the teaching of mathematics.

CANADA

From the reply sent by the Canadian Education Association

Introduction

Since Canada is a federal country, having ten autonomous educational systems, numerous variations exist among the programmes offered in schools in the different provinces. This becomes particularly apparent in attempting to prepare a report upon the teaching of a specific subject. In the following, therefore, the method adopted has been to consider as typical the situations most frequently encountered ; variations from this basic pattern have been disregarded, except where they are considered as of particular importance in the mathematics syllabuses of the provinces concerned.

Place of Mathematics in Curriculum

Any definition of the term " secondary education " for Canada as a whole must necessarily be arbitrary, since there are at the present time three distinct plans of organization of school systems. In some provinces the first eight grades (or years) of schooling constitute the primary school, with the succeeding four (or in two provinces, five) grades constituting the secondary school. Other provinces employ a 6-3-3 plan, under which the primary school terminates at the end of grade VI, with grades VII-IX inclusive as the junior secondary school, and grades X-XII as the senior secondary school. In one province (Quebec) there is a 7-2-3 plan, that is, seven years primary, two years junior secondary school, and three years senior secondary school. For purposes of simplicity in this report it has been assumed that the secondary school consists of grades VII-XII (or XIII). These grades correspond roughly to ages 13 to 18.

In the majority of provinces, mathematics is a compulsory subject to the end of grade IX and becomes wholly or partly optional in the senior secondary school grades unless the pupil is enrolled in a pre-university course. In the province of Newfoundland a somewhat exceptional situation exists in that arithmetic is compulsory in grades IX and X, but algebra and geometry are optional. In Manitoba, Prince Edward Island, and Quebec, mathematics is a compulsory subject throughout the secondary grades, with a partial exception in Quebec, where geometry is not required of girls in the general and commercial courses of the senior secondary school. It should be noted, however, that despite the frequently optional nature of the subject, in practice almost all pupils include at least one mathematics course in their senior secondary school work.

The proportion of time devoted to mathematics in relation to the rest of the time-table consequently varies from province to province and from grade to grade. In grades VII-IX it averages 12% to 16% of school

time per week. In senior secondary grades the proportion may run as high as 20% or as low as 12%, depending upon whether the student is enrolled in a matriculation (university preparatory), general academic, or vocational course. In grade XIII (Ontario and British Columbia), in the case of students who are planning to enter a scientific faculty of the university, as much as 36% of school time may be devoted to mathematics. In the Roman Catholic school system of Quebec there is a differential based on sex; in each year of the secondary school girls spend one hour less per week on mathematics than do boys.

Generally speaking, mathematics ranks equally with English and social studies in both promotion and school-leaving examinations.

Aims of Mathematics Teaching

Seven of the ten provinces include in their courses of study statements of the aims of mathematics teaching in the secondary school. Despite differences in detail, the underlying philosophy is sufficiently similar in all cases for these aims to be summarized as follows: the development of the ability to apply mathematical principles to the interpretation and solution of every-day problems; the further development of competence in, and understanding of, the basic mathematical skills, and the power to apply them in new situations; the development of habits of logical and critical thought and intellectual honesty; the development of a sense of the importance of accuracy, neatness, and precision; the development of an appreciation of the contribution of mathematics to the progress of civilization, and of the universal application of mathematics in science, commerce, industry, and architecture.

The above are general aims applicable to all grades. More specifically, the mathematics course in the junior grades of the secondary school aims to provide an opportunity for the individual student to discover his aptitudes and abilities and determine the part mathematics should play in his future school career, while at the same time strengthening the mastery of fundamental processes and providing a basic grounding in algebra and geometry for those who will not continue their formal education beyond the period of compulsory school attendance (the end of grade IX or X). At the senior secondary school level, mathematics teaching seeks either to prepare the student for advanced work at the university level or to provide thorough training in the practical uses of mathematics in commercial or technical fields.

Syllabuses

Grades VII and VIII. In general, the mathematics courses offered in grades VII and VIII in all provinces consist substantially of arithmetic, along with some introductory work in algebra and geometry. The questions treated in the arithmetic section of these courses include, typically, the following: review and expansion of fundamental operations with numbers; meaning, manipulation, and use of common, decimal, and percentage fractions; tables of measurement and their use; graphical work (mainly bar, line, and circle pictorial graphs); problems based on

home and business situations, stressing applications of percentage. The introductory work in algebra usually includes : the use of letters to represent numbers ; the elementary algebraic conventions regarding the expression of addition, subtraction, multiplication, and division of letters, and the use of parentheses ; introduction to equations ; introduction to the formula and its use in geometry problems. The introductory work in geometry generally includes : introduction of basic concepts regarding lines, angles, and circles ; constructional work, e.g., bisecting angles and drawing perpendiculars ; computation of perimeters and areas of common plane figures, using formulas when appropriate.

Grade IX. In general, the grade IX mathematics course continues the work in arithmetic, algebra, and geometry of grades VII and VIII, but with less attention to arithmetic and more attention to algebra and geometry. The arithmetic portion of the course includes a " maintenance programme " on the fundamental operations with whole numbers, fractions, and denominate numbers ; there is also some problem work, mainly a continuation of the grades VII and VIII problems based on home and business situations. The algebra questions treated include : elementary linear equations and their use in solving problems ; positive and negative numbers (their meaning and addition, subtraction, multiplication, and division) ; the formula (its nature and use in solving arithmetic and geometric problems) ; straight-line graphs ; first steps in factoring, and operations with algebraic fractions. The geometry includes : further constructions, including scale drawings ; special emphasis on accuracy in constructions ; basic concepts regarding angles, circles, and polygons ; special study of triangles : classification, construction, congruency (no theoretical proofs).

Grade X. At the grade X level, alternative mathematics courses are frequently offered (in a few provinces, these alternative courses are begun in grade IX) ; typically, there are three : (a) matriculation ; (b) commercial ; (c) technical. Students usually elect one of these depending upon the school programme which they are following. The content of the matriculation course generally consists of : a fairly substantial section of algebra, including further work in equations, fractions, formulas, and factorization ; a fairly substantial section of geometry, including theoretical proofs of theorems and problems concerning congruency of triangles, parallel lines and parallelograms, and angles in triangles and polygons ; a smaller section on arithmetic, normally a " maintenance programme ". The commercial course usually consists of business arithmetic, e.g., banking problems, invoicing, and profit and loss. The technical course usually consists of topics with applications in shop work, e.g., mensuration and constructional geometry.

Grade XI. The pattern of alternative courses is generally continued through this year. The matriculation course typically includes questions of algebra and geometry ; the algebra topics most commonly found include linear equations containing fractions, simultaneous linear equations, quadratic equations, more advanced factorization, indices and surds ; the geometry topics most commonly include theoretical proofs of theorems and problems concerning locus, circles, and areas. The commercial course

typically includes more advanced work on business arithmetic, e.g., buying and selling on commission, purchasing personal property, and interest and discount. The technical course typically includes application of arithmetic, algebra, geometry, and trigonometry to practical shop problems.

Grade XII. Alternative courses are not so common in this grade as in grades X and XI, although some provinces offer advanced courses in technical mathematics. Matriculation mathematics usually consists of two and sometimes three courses, comprising : more advanced algebra, including ratio, proportion and variation, series, binomial theorem, logarithms, irrational numbers, and fractions ; a first course in trigonometry including a study of the trigonometric functions and their relationships, the solution of triangles (both right-angled and oblique), radian measure, and trigonometric equations ; a first course in analytical geometry including theorems and problems on the straight line and the conic sections.

Grade XIII. This grade is offered in only two provinces. The mathematics taught is really first-year university mathematics, and covers advanced algebra, trigonometry, analytical geometry, and statics.

Mathematics syllabuses in Canada do not include any organized statements on the history of mathematics, although in some cases there are brief incidental references to outstanding historical events in this field. There is typically an introduction to axioms included in the first geometry course which dealt with theoretical proofs. Ordinarily this introduction is informal and not given a great deal of emphasis.

Methods of Teaching Mathematics

All provincial Departments of Education issue official curriculum bulletins or courses of study outlining prescribed courses and listing authorized textbooks. Usually these bulletins contain some instructions or suggestions concerning the teaching methods to be used in mathematics, but these are often quite brief, sometimes taking the form only of a few general suggestions stressing the importance of such matters as motivating the class or teaching for understanding. Occasionally, they contain specific teaching suggestions for certain topics. On the whole, however, the major responsibility for acquainting teachers with proper methodology is left to the teacher-training institutions.

In teaching new topics, the method most commonly employed in the secondary school mathematics classroom is the "modified lecture" method. The instructor "lectures" in the sense that he analyzes and explains the topic to his class, but at the same time he uses the blackboard (or occasionally other teaching aids) for purposes of illustration or demonstration, questions the pupils frequently to test their understanding, and invites them to ask him questions if they have difficulty in following his exposition. Different methods are used on other occasions. For example, during a period devoted to mastering more completely a newly-learned topic, the students are often asked to work individually at a series of progressively more difficult practice exercises, thus leaving the teacher free to encourage and assist individual students.

There are no special instructions on teaching methods for mathematics at the transfer stage from primary to secondary school, except in a few instances where some of the general teaching suggestions in official curriculum bulletins may touch on this matter. It is not uncommon to find brief mention in provincial courses of study of the desirability of coordination between the teaching of mathematics and other subjects in the secondary school. The actual implementation of this idea, however, rests largely in the hands of individual schools and teachers. It is fairly common for the mathematics instructor to be assigned also to teach some of the sciences, such as physics or chemistry. This places him in a favourable position for achieving co-ordination among the group of subjects which he teaches. The extent of the coordination achieved, of course, depends largely upon the interest and skill of the individual teacher. Encouragement on the part of local supervisors can also be a significant factor.

Instructional aids such as films are not widely used in the teaching of mathematics in Canadian secondary schools, although there are exceptions in the case of individual teachers. The teaching aids most commonly employed are models of geometric forms and illustrative charts. Typically, however, mathematics teachers rely mainly on the blackboard and on the common geometrical instruments, such as protractor and compass.

The mathematics syllabuses included in technical and commercial courses provide the most notable examples of applied mathematics in secondary schools ; stress in these is laid upon the use of mathematics in the solution of practical problems. In matriculation courses, although the immediate application of mathematics is not the primary objective, teachers take advantage of such opportunities as may arise to show students how the various skills may be used towards a better understanding and control of their environment.

In all provinces the textbooks to be used in teaching secondary school mathematics are prescribed by the provincial Department of Education, although in some provinces local school systems are given a choice of books from a prescribed list.

Teachers

Generally speaking, teachers of any subject in Canadian secondary schools, particularly at the senior secondary school level, are required to possess a university degree plus one or more years of professional training. In the majority of schools the mathematics teacher is not required to hold special qualifications beyond this. The local education authorities, however, who are responsible for the selection of their own teachers, endeavour, where an applicant may be called upon to instruct classes in mathematics, to obtain one having some special interest in the subject. In the large secondary schools of the main urban centres teachers of mathematics are usually required to be " specialists "—that is, persons who hold either an arts degree with honours in mathematics or a degree in science, as well as the proper teaching certificate for the grades they teach. In junior secondary schools and in small rural secondary schools it is sometimes found necessary to employ teachers who have had only one or two years of university training.

Since any qualified secondary school teacher may be called upon to give instruction in mathematics, there appears as yet to be no greater difficulty in recruiting non-specialist teachers than is consequent upon Canada's overall shortage of teachers. There is, however, a distinct shortage of specialists, since many students of outstanding mathematical aptitude are attracted towards business and industry because of the greater financial rewards in these fields. So far as remedial measures are concerned, the steps taken in connection with the general shortage of teachers apply equally to teachers of mathematics; these include bursaries and loans to assist teachers-in-training, increased salaries, improved pension plans, and better working conditions. Supplementary allowances on salary scales are paid to specialist teachers of mathematics as to specialists in other subjects.

Little special consideration has been given to keeping mathematics teachers up-to-date in their knowledge. Voluntary in-service summer courses in mathematics as in other subjects are offered at universities in most provinces. In a few provinces, small grants are available to encourage any teacher to improve his professional qualifications through attendance at summer school. In Nova Scotia and New Brunswick an annual summer school for mathematics teachers is sponsored by the Canadian Mathematical Congress. Ontario has a provincial organization of teachers of mathematics and physics and in the same province the professional association of secondary school teachers is developing subject-study groups. Interchange of teachers is limited to the interprovincial and international exchanges arranged by the Canadian Education Association; although these may occasionally involve the exchange of mathematics teachers, this is incidental, the exchanges being of a general nature over the whole field of primary and secondary education.

Miscellaneous

No major modifications in the teaching of mathematics are reported by the provinces as being in prospect at the present time. The purpose of mathematics courses at the secondary school level is, as has been stated, to provide a thorough grounding in the fundamentals, to serve as a foundation for more advanced study at the university, or to meet practical needs in post-school life. The subject is thus, by its very nature, less likely to undergo extensive modifications in course-content or methods of instruction than are subjects of greater flexibility and less clearly-defined purpose, such as social studies or art. As with all other subjects, however, mathematics syllabuses are appraised at intervals of from three to five years, with the aim of adapting them to developing needs. An illustration is the introduction in recent years in several provinces of separate mathetics courses designed to meet the differing needs of students proceeding to university and those planning to enter employment upon graduation from secondary school.

Because of Canada's small population, the market for educational books in specialized fields is very limited. In consequence, there are few, if any, Canadian texts on methods of teaching mathematics in secondary schools. Canadian teachers seeking information on this subject rely mainly upon publications originating in the United States or Great Britain.

CEYLON

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Secondary education comprises a three-year junior cycle, a two-year senior cycle, and a two-year collegiate cycle. The age range of the pupils in the junior cycle is from 11 to 14 years. Four periods a week are allotted to mathematics in the junior cycle, and five periods in the senior cycle, excluding five and four additional periods a week for arithmetic in the junior and the senior cycle respectively. Nine periods a week in all is thus allotted to mathematics in both the junior and the senior cycle, about 20% of the total time-tables.

Mathematics embraces arithmetic, geometry and algebra, and, in the senior and the collegiate cycle (in which pupils specialize in mathematics) advanced and applied mathematics. Arithmetic is a compulsory subject in all schools. Mathematics was often optional in what were formerly the English-medium schools, and is hardly taught in the national-language *(Swabasha)* schools.

No special importance is attached to mathematics, in comparison with other subjects.

Syllabuses

It should be noted that, with regard to the arithmetic syllabuses, teachers may follow any sequence of topics they think proper, and are not required to follow that given in those syllabuses. All the money problems they set, except those involving exchange transactions, should be based on Ceylon currency. The syllabuses suggest some applications of arithmetic to real life situations, but this has been left largely to the initiative of the teacher. Children should always be encouraged to ask themselves if their replies are reasonable.

1st year (standard VI). Arithmetic : a review of the fundamental processes, and their application to Ceylon money, and of the units of measurement ; factors and multiples, vulgar and decimal fractions, the unitary method ; the area of rectangles, volume, averages ; problems. Geometry : basic ideas, angles, parallel lines triangles (theorems and diagrams), the circle, ruler and compass constructions. Algebra : the relationship between arithmetic and algebra, the use of symbols, problems leading to simple equations, positive and negative signs. — *2nd year* (standard VII). Arithmetic : continuation of the previous year's work ; ratio and proportion, percentages, simple interest, rates and taxes, square root, the metric system, English and Indian money. Geometry : the isosceles triangle, four-sided figures, the area of a triangle, right-angled

triangles, further work on the circle. Algebra : continuation of the previous year's work ; simultaneous equations with two unknowns, fractions, easy factors ; graphical representation of tables of statistics, and the reading of graphs. — *3rd year* (standard VIII). Arithmetic : revision and continuation of work done in previous years ; problems on time, work, and distance (including relative velocity) ; graphs of statistics (geographical and economic data), temperature charts, etc. Geometry : revision of the work done in previous years ; the theorem of Pythagoras with practical illustrations, divers theorems and exercises, loci. Algebra : revision of the work done in previous years ; factorization (difference of two squares, sum and difference of two cubes, etc.), problems leading to quadratic equations, graphs (including the solution of simultaneous equations).

The central schools algebra syllabus is as follows : *1st year* (standard VI) : relationship between arithmetic and algebra, use of symbols, the four opérations. — *2nd year* (standard VII) : problems leading to simple equations and later to simultaneous equations, fractions, graphical representation of tables of statistics and reading of graphs. — *3rd year* (standard VIII) : revisions of previous year's work, difference of two squares, problems leading to quadratic equations, graphs relative to the solution of simultaneous equations.

Methods of Teaching Mathematics

There are no official instructions concerning methods of teaching mathematics at secondary level, nor are there any official instructions concerning the transition stage from primary to secondary education. The importance of correlation between mathematics and other subjects in the secondary curriculum is generally stressed in the mathematics courses in all the teacher-training schools, and has also received emphasis through the introduction of handicrafts into the schools.

Hardly any of the aids mentioned in the questionnaire are used in the teaching of mathematics, but models are often used in the teaching of geometry and algebra. Practical work offers an attractive approach to the teaching of mathematics. Applied mathematics is taught only in the senior cycle. Activity methods have been introduced into the teaching of mathematics as a result of the introduction in recent years of handicrafts based on projects.

School textbooks are approved by an educational publications board.

Teachers

Mathematics is taught in secondary schools by university graduates or very experienced trained teachers.

There is a great scarcity of mathematics teachers capable of teaching in the national languages. This is due to the fact that mathematics was taught only in the English-medium schools, and that university graduates offering mathematics do not often become teachers, as they have more lucrative avenues in other fields. To overcome this scarcity, specialist

courses have been started in a secondary-teacher training college, and mathematics has also been introduced into all the national-languages teacher-training schools to enable trainees who have never learnt any mathematics to begin to do so. Vacation courses are organized to keep mathematics teachers up-to-date in their subject.

Miscellaneous

The question of modifications in the mathematics syllabuses is now under consideration by a committee appointed by the Ministry of Education to draw up a curriculum suited to the needs of the Ceylon schools.

No books on the teaching of mathematics in secondary schools have been published in Ceylon, but in recent years attempts have been made to familiarize teachers in the national languages with modern methods of teaching the subject.

CHILE

From the reply sent by the Ministry of Public Education

Place of Mathematics in Curriculum

Secondary education in Chile lasts six years. Children enter a secondary school at the age of eleven plus, and normally sit for the baccalaureate examination at the age of about eighteen. Under the designation of *matemáticas*, mathematics is taught as a compulsory subject throughout the course. The number of hours per week allocated to it are as follows (the figure in brackets is the total number of hours in the weekly time-table) : 1st year, 5 (31) ; 2nd year, 4 (34) ; 3rd year, 4 (37) ; 4th year, 3 (36) ; 5th year, 3 (36) ; 6th year : 3 (scientific section) ; 3 hours for mathematics and physics (arts section).

The regulations on secondary education, moreover, make it possible for a minimum of ten pupils in the scientific section, 6th year, wishing to make a more advanced study of mathematics, to take an extra hour in the subject, and to do one hour less on science.

Mathematics is one of the subjects to which most time is allocated in the time-table, but is not more important than other subjects in promotion or school leaving examinations.

Aims of Mathematics Teaching

Mathematics is regarded as an integral part of secondary education, and as contributing to the realization of the general aims of secondary education, which are to develop the pupils' personality to the full, and to prepare them for citizenship and participation in economic activities, without claiming to give them any truly vocational training. The subject is also considered to have its own specific aims, which are as follows : to accustom the pupils to formulating, analysing and interpreting problems, and to collecting and utilising the data required for their solution ; to develop the pupils' capacity to think in a way enabling them to comprehend quantitative relationships in nature, production, and everyday life ; to develop the pupils' capacity for deduction in relation to all phenomena involving groups of inter-related factors ; to enable the pupils to understand and employ mathematical symbols ; to enable them to appreciate the part played by mathematics in science and technology ; to assist the development of the pupils' power of logical reasoning, accuracy in the use of words, precision, truthfulness, and industry.

Syllabuses

The mathematics syllabus for the three-year, lower secondary course covers arithmetic (decimal system, fractions, rule of three, percentages,

commercial arithmetic, averages mixtures, alloys), plane geometry, and (in the 3rd year) elementary principles of algebra.

The mathematics syllabus for the upper secondary course covers continued work in algebra and geometry, and (in the 6th year, in both the science and the arts section) cosmography.

The syllabuses make no specific reference to the history of mathemathics, but teachers may do so on occasion.

Methods of Teaching Mathematics

The official mathematics syllabuses contain suggestions on methods of teaching the subject. Teachers are recommended above all to encourage their pupils' capacity for conscientious and independent work, or, in other words, to adopt the activity approach.

When drawing up the mathematics syllabuses, the authorities were concerned to ensure the continuity of primary and secondary education, and close liaison between mathematics and physics.

In most secondary schools, textbooks approved by the Ministry of Education are used for the teaching of mathematics. It is of interest to note, however, that seven of the secondary schools are working on experimental lines, and have teaching materials prepared by the *Sección Perfeccionamiento y Orientación de la Dirección general de Educación secundaria.*

Teachers

Secondary teachers of mathematics are trained at the institute of education in the faculty of philosophy and education of the University of Chile. The course is a five-year one, and concludes with the award of the title of *Profesor de Estado.*

Further training is organized for teachers throughout the country by the *Perfeccionamiento y Orientación* section of the secondary education directorate. The same body also gives advice and guidance to teachers, through circulars issued with a view to the improvement of teaching methods. Teachers also have opportunities of securing further training through the libraries in the secondary schools, a periodical *(Revista de Educación)* published by the Ministry of Education, and scholarships granted by the government or by cultural bodies abroad.

Miscellaneous

In consultation with the general inspectorate *(Superintendencia de Educación)*, the secondary education directorate is at present working on a revision of the secondary curriculum, and a number of special committees have been appointed to that end.

CHINA (Republic of)

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught throughout the six years of secondary education, which is divided into a lower and an upper cycle, each lasting three years. The age range of the pupils is from 12 to 18 years. Three hours a week are allotted to the subject in the lower cycle, and four hours in the upper cycle, representing about 10% of the total time-table.

Mathematics embraces arithmetic in the 1st year, algebra in the 1st, the 2nd, the 3rd, the 5th and the 6th year, geometry in the 2nd, the 3rd, the 4th and the 5th year, trigonometry in the 4th year, and an introduction to analytic geometry in the 6th year.

Mathematics is a required course. In promotion examinations the same importance is attached to mathematics as to other subjects. A pass is required in mathematics in the school leaving examinations. The latter examinations are more difficult than the promotion examinations.

Aims of Mathematics Teaching

The aims of mathematics teaching in the lower cycle are to lead the pupils to understand the nature of the figure and the number, the relationships between them, and the methods and general principles of mathematics, to give the pupils knowledge relating to their daily life, and to develop their reasoning and their power to find the unknown from the known.

The aims in the upper cycle are to show the pupils the inter-relationships between algebra, geometry and trigonometry, to train them in the use of the deductive method, to give them the mathematical knowledge necessary for the study of other subjects, to develop their capacity for practical calculation and drawing, and their analytic ability, inductive reasoning, and aptitude for research, and, finally, to lead them to appreciate the usefulness and wide field of application of mathematics, and thus to stimulate their desire for further study.

Syllabuses

The secondary mathematics syllabuses are now under revision.

The syllabuses include the history of mathematics and an introduction to axioms.

Methods of Teaching Mathematics

Instructions concerning methods of teaching mathematics are issued by the Ministry of Education. Such teaching is required to be conducted as comprehensively as possible. Stress is laid on the question of the relationships between the number and the figure; the figure is more frequently employed for illustration in algebra, while in geometry more use is made of algebraic proof. In the lower cycle the main emphasis should be on calculation.

In the lower cycle, in order that the pupils should have a full understanding of the fundamental ideas of mathematics, moreover, it is more necessary to do practical exercises than to learn abstract theories, and such exercises should be related in every possible way to the pupils' daily life. Given the disparity of aptitude for mathematics, pupils under the average in ability should be given individual attention, and those of higher ability should be given extra work.

In the upper cycle, the teaching methods are similar to those employed in the lower cycle so far as lectures, discussions, exercises, assignments, tests and reviews are concerned. The reasoning power of the pupils, however, is naturally higher, and it is therefore advisable for the teacher to assign material in advance and to have the pupils study it before class, and also to have them explain simple questions, rather than doing so himself, and make a summary or diagram on completing a given chapter or section.

In the upper cycle, furthermore, reference books and periodicals are assigned to the pupils as materials to be studied either individually or in a group, and reported on. The pupils are advised to keep a notebook. If time permits, geometric drawing, surveying, statistics, etc., may be taught as additional courses.

There are no special instructions concerning the transition from primary to secondary education, but in this connection teachers may refer to the secondary curriculum issued by the Ministry of Education. As to the correlation between mathematics and such subjects as physics and economics, the textbooks for the various subjects in the secondary curriculum are carefully drawn up in accordance with that curriculum, and duplication is avoided.

The teaching materials for mathematics lessons generally comprise wall charts. Preparations are being made to make slides, etc., for mathematics teaching.

Practical work and experiments are given in all secondary schools except the few where the necessary facilities are lacking.

Schools are required to adopt the standard textbooks as approved by the Ministry of Education.

Teachers

To be officially qualified, a secondary teacher of mathematics must be a graduate of the mathematics department of the higher school of teacher training (Taiwan Provincial Normal University).

Some difficulty is experienced in recruiting mathematics teachers for secondary schools in rural areas. This is due to the fact that the number of secondary schools increases steadily, and that relatively few university students take mathematics as their major field of study. To remedy the shortage teachers with ability and experience in the teaching of mathematics are appointed as substitutes.

Each secondary school has a mathematics association organized by the teachers to improve their knowledge. The Taiwan provincial higher school of teacher training has sponsored a programme of summer vacation seminars for secondary teachers.

Miscellaneous

As noted above, the revision of the secondary mathematics syllabuses is under consideration. The modifications envisaged concern mainly those parts of the syllabuses which are not sufficiently related to the daily necessities of life.

Theses on methods of teaching mathematics have been published in various journals. There is at present no publication specially devoted to this subject.

COLOMBIA

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject during the first five years of secondary education (pupils usually from 13 to 17 years of age). The subject is designated *aritmética* in the 1st and the 2nd year (the time allocation being five hours out of a total of twenty-four hours a week), *álgebra y contabilidad* in the 3rd year (the time allocation being five hours out of a total of twenty-three hours a week), *álgebra y geometría* in the 4th year (the time allocation being seven hours out of a total of twenty-five hours a week), and *geometría del espacio y trigonometría* in the 5th year (the time allocation being three hours out of a total of twenty-five hours a week).

The same importance is attached in the secondary curriculum to mathematics as to the other academic subjects.

Aims of Mathematics Teaching

The aims of teaching mathematics may be grouped under the following heads. (a) *Aims relating to character formation.* Mathematics inculcates habits of reflection, attention, verbal exactitude, and concentration, develops reasoning powers, and gives ample mental exercise ; the subject, in short, enables pupils to form moral habits, including orderliness, objectivity, exactitude, intellectual honesty, and firmness of character. (b) *Aims relating to mental development.* The close reasoning that mathematics calls for, greatly assists intellectual growth. (c) *Cultural aims.* A study of mathematics serves cultural aims, since it makes a considerable contribution to the stock of knowledge a pupil carries over into his adult life. (d) *Practical aims.* Mathematics forms a basis for the study of other subjects such as physics and chemistry, and of certain professions (engineering and allied professions), and is also of considerable practical value in everyday life.

Syllabuses

1st year. Exercises on the four fundamental operations (whole numbers, vulgar fractions, decimal fractions) ; the decimal system of measurements and its application to mensuration ; application to harder numerical calculations ; rule of three. — *2nd year.* Ratio and proportion ; continued proportion of three ; proportional parts ; percentage, interest, discount, foreign exchange, averages, alloys. — *3rd year.* Introduction to algebra, operations with algebraic numbers, highest common factor,

fractions, ratios, first-degree equations in one unknown, systems of equations. — *4th year.* Second-degree equations, arithmetical and geometrical progressions, logarithms, compound interest. Plane geometry: introduction, straight lines, parallels, etc.; polygons, the circle, similar polygons, areas. — *5th year.* Introduction to trigonometry, fundamental properties of triangles, angles, solution of triangles. Geometry in space: relative positions of lines and planes in space; polyhedrons, round bodies.

The syllabuses make no reference to the history of mathematics or to the principles of axioms. On suitable occasions, however, the teachers themselves may refer to the history of mathematics.

Methods of Teaching Mathematics

Teachers of mathematics are free to choose whatever method they prefer of teaching the subject. Generally speaking, they endeavour to lead their pupils to a logical elaboration of ideas, through both induction and deduction, their main concern being to enable their pupils to be objective in regard to both concrete and abstract matters, and to arouse their interest. Their approach is fundamentally different to that of their colleagues in primary schools, where the teacher's dominating position tends to make teaching methods authoritarian.

The materials used for mathematics lessons consist of pictures illustrating the decimal system of measurements, and set-squares, compasses, rulers, and plane and three-dimensional geometrical figures.

The teaching of mathematics is closely coordinated with that of physics; the experiments and the problems of physics involve arithmetical calculations, and the laws of physics find expression in algebraical formulas.

The theoretical ideas gained through arithmetic, geometry, and algebra are applied practically in the solution of various concrete problems relating to industry and commerce, surveying, and the making of drawings and diagrams.

To a certain extent teachers of mathematics make use of activity methods. In the initial stages, for example, they use concrete materials (objects, plasticene, cardboard, wood) to explain fractions, measuring instruments to explain the decimal system of measurements, etc., and drawings and diagrams, and only gradually go over to abstract ideas. They also give their pupils practical exercises on the use of a theodolite, in trigonometry, and on making three-dimensional figures, in geometry, and thus secure their active participation.

Teachers of mathematics are also free to choose what textbooks they prefer. In any case the textbook is regarded simply as a guide, and the pupils supplement and complete it with annotations, and explanatory notes in their exercise books. Some teachers do without a textbook, and the pupils then take notes during the year, and so make up a kind of book of their own.

Teachers

Teachers of mathematics at secondary level are required to hold the baccalaureate or the diploma of complete teacher training (six-year course),

and then to have completed the four-year specialist course at the Pedagogical University, covering the following subjects : integral and differential calculus, differential equations, differential, analytical and descriptive geometry, theory and practice of education, psychology, philosophy, school hygiene, history of art, advanced physics, and methodology. Through this course teachers of mathematics thus receive both their general and their professional training. Having obtained his first degree, and taught for two years in a secondary school, a teacher is permitted to submit a thesis for the doctorate.

These requirements are not often met, however, in actual practice. The persons holding them can obtain better paid posts than teaching posts. Ways of remedying the resulting difficulty in recruiting sufficient teachers with these requirements, are now being sought.

Miscellaneous

No changes are likely to be made in the near future in the way in which mathematics is at present taught ; it has not been greatly influenced by modern developments.

Articles appear from time to time on the methods of teaching mathematics, in scientific journals, and lectures are given on them in the teacher training establishments. Several mathematics textbooks have been published recently.

COSTA RICA

From the reply sent by the Ministry of Public Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the five-year course of secondary education. The age range of the pupils is from 13 to 17 years. Four hours a week is allotted to the subject, which is 11.11% of the total time-table. The subject is designated *matemáticas;* the same importance is attached to it as to other subjects in the secondary curriculum.

Aims of Mathematics Teaching

In conformity with the principles underlying the new syllabuses, the main aims of mathematics teaching are to develop the research attitude, the habit of reaching the solution to a problem through an analysis of its data. The enunciation and proof of theorems are not regarded as aims in themselves, for the reason, given in the suggestions concerning the syllabuses, that " one is not asked in practice to prove a theorem, but to solve a problem ; life often confronts us with problems." The mathematical mentality is part of the attitude of research, and must therefore be regarded as an instrument which pupils will find useful in practical life.

Syllabuses

1st year. Numeration, the fundamental operations and problems of application, raising to a power, the nature of divisibility, fractions, ratios, proportions, the rule of three, the decimal system of measurements and old measures. — *2nd year.* Introduction to algebra, algebraical symbols, positive and negative numbers, algebraical expressions and the corresponding operations, products of algebraical expressions, the highest common factor, operations on algebraical fractions, equations. Geometry : introductory notions, angles. — *3rd year.* Systems of equations with two or three unknowns, powers and roots, square root. Geometry : problems concerning the circumference. — *4th year.* Second-degree equations, relations between the coefficients and the roots of a second-degree equation, logarithms, arithmetical and geometrical progressions, compound interest. Geometry : regular polygons (construction), similar polygons. — *5th year.* Trigonometry : a study of the dimensional relations among the elements of a triangle, the solution of right-angled triangles, trigonometrical functions, sine, cosine, tangent, application of logarithms.

The syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

Suggestions as to the methods of teaching mathematics were drawn up at the same time as the official syllabuses. Teachers are recommended to give their pupils only those problems in which the theorems proved find a practical application, and not to ask them to learn by heart the proof, but to impart to them a research spirit whereby they will discover for themselves mathematical relationships and laws. In regard to the proof of theorems, moreover, teachers are recommended to give the simplest and most immediately comprehensible one. It is of no use to get the pupils to repeat the enunciation of a problem, since it does not help them to understand the work ; they should be asked to analyse the data given and to identify the unknown data to be determined. Pupils should also be asked to find new problems themselves for the application of theorems, and to present them for solution by the class as a whole.

The use of only one exercise book is also recommended, grouped in the same order as the baccalaureate examination syllabus.

No special instructions or suggestions have been issued concerning the teaching methods to be used during the period of transition from primary to secondary education.

The education authorities are at present making a careful examination of textbooks with a view to the selection of the best of them.

Teachers

No special arrangements have been made for the professional training of secondary teachers of mathematics. Preference is given to candidates who have completed courses in engineering or hold a diploma from the institute of education.

Some difficulty is encountered in recruiting secondary teachers of mathematics, on account of the insufficiency of the salaries offered. The government, however, has recently decided that all teachers' salaries must be raised.

In collaboration with the university the education authorities are now examining a scheme of further training for secondary teachers.

Miscellaneous

A thoroughgoing reform of the secondary course, including mathematics, is envisaged.

CUBA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Under the designation of *matemáticas*, mathematics is taught as a compulsory subject throughout the first four of the five years of secondary education, and throughout the fifth year for some pupils. The age range of the pupils is from 12 to 17 years. Five hours a week (25% of the total time-table) are allotted to the subject in the 1st and the 2nd year, three hours (15%) in the 3rd and the 4th year, and five hours (25%) in the 5th year (in this year the subject is compulsory only for the pupils in the science section preparing for the baccalaureate examination, and is designated *ampliación de matemáticas*).

The same importance attaches to mathematics as to other subjects in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The main aim of teaching mathematics is to impart knowledge of practical usefulness ; emphasis is given to the fundamental notions, and care is taken to avoid overloading the pupils with too much theory.

Syllabuses

1st year. The fundamental operations on whole numbers, fractions and decimals, problems involving percentages, simple interest, discount, and proportional parts. Algebra : fundamental notions, simple equations. — *2nd year.* Algebra : common factors, operations on algebraical fractions, first-degree equations, systems of two or three equations. Geometry : an intuitive study of the fundamental notions, a study of some theorems and their practical applications to angles, triangles, polygons, and the circumference. — *3rd year.* Algebra : roots, equations of the second degree with one unknown, progressions, logarithms, compound interest. Geometry : cases of similarity of triangles, dimensional relations of the elements of the triangle, notions of trigonometry. — *4th year.* Geometry : dimensional relations in the circle and in regular polygons, areas of plane figures with finding of the formulas, polyhedrons and round bodies, areas and volumes. Trigonometry : fundamental formulas, solution of triangles. *5th year, science section only.* Additional work in arithmetic, algebra, and geometry ; irrational numbers, properties of whole polynomials with one variable, complex numbers, systems of indeterminate equations and inequalities, elementary properties of conic sections, analytical geometry.

Teaching with regard to the history of mathematics is limited to a brief account of the origin of the various branches of mathematics.

Methods of Teaching Mathematics

Teachers are recommended by the committee responsible for drawing up the syllabuses to avail themselves of all possible means of helping their pupils to understand mathematical ideas, by appealing to intuition when necessary, but without neglecting the demonstrations forming part of the rational method.

When the syllabuses were being drawn up, care was taken to correlate mathematics with other subjects in the curriculum, physics in particular. Teachers, moreover, are recommended always to choose problems showing the practical applications of mathematics.

As preparation for formal teaching, intuition is used rather than activity methods proper. The pupils do experimental work in physics.

The teaching materials used for mathematics lessons consist mainly of wall charts of units of measurement, and collections of geometrical solids. So far as textbooks are concerned, teachers are free to choose those they prefer; many teachers have themselves written mathematics textbooks.

Teachers

Secondary teachers of mathematics are required to hold the doctorate in science, preference being given to those who have specialized in physics and mathematics. No difficulty is experienced in recruiting them.

Secondary teachers may obtain scholarships for one year's study abroad.

Miscellaneous

No modifications are at present envisaged in the teaching of mathematics, but the Ministry of Education proposes the organization in the near future of seminars in this connection.

Some account was taken of modern developments in the mathematical field when the mathematics syllabuses now in force were being drawn up.

CZECHOSLOVAKIA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught throughout the six years of secondary education (classes VI to XI inclusive of the general education schools). The age range of the pupils from 11 to 17 years. Throughout the course mathematics is designated as such. Mathematics is compulsory in all years. The time allocated is seven hours a week (22.6% of the total number of hours) in the 1st year (class VI), six hours (18%) in the 2nd year (class VII), five and a half hours (17%) in the 3rd year (class VIII), and six hours (17.6%) in the 4th, the 5th and the 6th year (classes IX-XI).

The same importance is attached to the subject as to the mother tongue ; these two subjects form the basis of general education. Mathematics is included in the examination coming at the end of the 3rd year (class VIII) and in the matriculation examination coming at the end of the 6th year (class XI).

Aims of Mathematics Teaching

Mathematics is regarded as having both a general instructional and educational aim and a special aim. Its purpose, that is to say, is to give the pupils basic theoretical and practical knowledge, to cultivate their capacity to think logically, scientifically, and abstractly, and to prepare them for entry into a profession or technical studies at higher level.

Syllabuses

Mathematics as taught at secondary level comprises arithmetic, algebra, geometry, and trigonometry.

Arithmetic covers whole numbers and fractions (including decimal fractions and percentages), and the application of the knowledge acquired to the solution of problems and simple everyday calculations.

Algebra comprises the concept of number (rational, real and complex numbers), functions, the solution of equations and inequalities, the notion of the limit, and the application of the knowledge acquired to the solution of simple problems relating to various branches of science and technology.

Geometry comprises a systematic study of the properties of plane and solid geometrical figures, the application of the knowledge acquired to problems involving calculation and construction, and practical activities, including work in the open air.

Trigonometry covers trigonometrical functions and their properties, the solution of triangles, and practical application of trigonometry in everyday affairs, geometry, physics, etc.

No time is specifically allocated to the history of mathematics, but it is referred to in the mathematics syllabuses and the textbooks. Instruction in axioms is given gradually over a number of years, but more especially in the course of the steriometry lessons in class X.

Methods of Teaching Mathematics

Each mathematics textbook has a companion handbook for teachers, explaining the system and distribution of subject-matter in the textbook and making recommendations as to methods. The teaching of mathematics is also greatly influenced by the journal *Matematika ve škole* (mathematics in school), whichis de voted to questions of method, and gives concrete examples of teaching procedures.

The essential problem is to combine, in a balanced way, the development of logical capacity with the acquisition of the skills required for numerical calculation and graphical work. Equal importance is attached to the scientific bases of mathematics, the formation of clear concepts, the precise use of axioms, definitions and theorems, and the deductive aspects of mathematics, on the one hand, and to learning how to apply knowledge, on the other.

The existence of comprehensive schools covering from the 1st to the 11th year of schooling ensures integration and harmony throughout the mathematics course.

Care was taken in drawing up syllabuses and textbooks to integrate mathematics with other subjects, physics in particular. Anomalies do occur, however ; the physics syllabus of the 4th year (class IX), for example, calls for mathematical knowledge of goniometric functions which the pupils will acquire only later. To remedy this, the physics teacher gives a summary explanation of the indispensable notions.

A minimum list of the materials required for mathematics lessons has been drawn up and published by the national educational publications service. Schools which have been in existence for some years are well equipped in this respect ; the newer schools are to be brought up to the same standard.

No films based on the new approach to mathematics have so far been produced, but their production is envisaged ; work has already begun on a film concerning geometry as a whole.

The new syllabuses give emphasis to practical measurement in the school and in the open air : topographical work, measuring distances, use of instruments, and technical drawing.

Teachers

The teachers of mathematics in the 1st, the 2nd and the 3rd year (classes VI to VIII) are trained for three years at a normal school, which they enter on completing the eleven-year school. The teachers of mathematics in the remaining three secondary years take the four-year course of a higher school of education or the five-year course of a science faculty. In all cases such training covers general and professional training and teaching practice. The training course concludes with a State examination.

There is an appreciable shortage of mathematics teachers. This has a number of causes; the demand for them has increased as a result of the increase of general-secondary, vocational and higher schools; pupils with mathematical talent prefer to study technological subjects, as these, in view of the country's rapid industrial expansion, offer them more scope.

In order to remedy the shortage of qualified secondary teachers of mathematics, the higher schools are beginning to organize correspondence courses for primary teachers, covering the entire university course of studies.

Teachers of mathematics may secure further training in a number of ways. Regional further training institutes, directed from a central institute, are opened successively in the various areas of the country. These regional institutes organize individual study, courses, practical work, conferences, and teaching practice. Meetings of primary and secondary teachers of mathematics, at district or regional level, are convened periodically. Articles on mathematical matters appear in the official journal of the Ministry of Education, *Matematika ve škole*, already mentioned, and to which all schools and a large number of individual teachers subscribe.

Miscellaneous

New mathematics syllabuses and textbooks were introduced at the beginning of the school year 1954-1955. Information is being collected as to the suitability of the textbooks, and to discover in what respects they may need modification. It appears that more importance will have to be given to practical calculations, and that it will be necessary to integrate mathematics more closely with physics.

The modern approach to mathematics finds full expression in the new textbooks. They give considerable emphasis, for example, to functions.

The suitability and degree of difficulty of the new textbooks are being investigated by the institute of educational research, and the findings will be published.

DENMARK

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Under the designation of algebra, geometry and arithmetic, mathematics is taught as a compulsory subject in the four years of lower secondary education (the 6th to the 9th year of schooling inclusive, pupils from 11 or 12 to 15 or 16 years of age). Four hours a week is allotted to the subject in the 1st year, five hours in the 2nd year, and six hours in the 3rd and the 4th year. In the *realklasse* (the 10th year of schooling, pupils of 16 or 17 years of age), five hours a week is allotted to algebra and arithmetic (optional for girls), and two hours a week to geometry (optional for both sexes).

Mathematics is also taught, under that name, as a compulsory subject in the three years of upper secondary education (the 10th to the 12th year of schooling, pupils from 16 to 18 years of age) in the science section. Six hours a week is allotted to it. The time allocation for mathematics in secondary schools represents from 12.5% to 20% of the total time-table.

Pupils leaving the lower secondary school and wishing to enter the science section of the upper secondary school, must reach a certain standard in mathematics. There are written and oral papers in mathematics in the school leaving examination.

Aims of Mathematics Teaching

In the lower secondary classes and the *realklasse*, mathematics is expected to teach pupils really to understand the rules governing the solution of problems, and carefully to examine the data available, or, in other words, to reason, and also to enable the pupils to apply the rules of arithmetic and to calculate with confidence and facility. The aim of the teaching is to develop the pupils' capacity for logical reasoning, and to enable them to express their conclusions briefly and accurately and to grasp the difference between the general and the particular.

In the upper secondary classes, the aim of mathematics is to impart fundamental mathematical knowledge, and to accustom the pupils to rigorous reasoning, precision, and accuracy in speech.

Syllabuses

Lower secondary classes. A study of elementary mathematics, as far as the extraction of square roots and the solution of biquadratic equations. Exercises on calculations (fractions, etc.) involving common everyday matters, such as insurance, interest, foreign exchange, etc. Geometry:

the calculation of areas and volumes, a study of the elements of geometry and of plane figures, as far as the proposition of similar triangles and the theorem of Pythagoras. — *Realklasse.* Polynomials of the second degree and common factors, equations of the first and the second degree with two unknowns, powers and roots, logarithms, compound interest, arithmetical and geometrical progressions. Geometry : similar figures, trigonometry, geometrical constructions. — *Upper secondary classes.* Algebra, trigonometry, differential and integral calculus, rational functions, trigonometrical functions, sine, cosine, tangent, cotangent, logarithmic and exponential functions. Geometry : plane and solid geometry, analytical geometry (conic sections).

Teachers are recommended to deal with those aspects of the history of mathematics, which are of general interest and most characteristic of each period.

Methods of Teaching Mathematics

There are no prescribed methods of teaching mathematics. The methods used by the teachers are those they have learned during their professional training.

The teaching methods used, moreover, are very similar to those used at primary level. A fair degree of continuity is thus achieved in the transition from primary to secondary education.

The teaching of mathematics is correlated with that of physics and chemistry. This is mainly effected by the care taken by the teachers to see that the mathematical knowledge needed for given work in these two subjects is imparted in advance of that work.

The principle materials used for mathematics lessons are the blackboard and geometrical solids. No use has so far been made of films. Apart from those for the teaching of science, no practical applications are prescribed, but teachers are free to introduce them on their own initiative.

In harmony with activity principles, frequent use is made of practical exercises at lower secondary level, as preparation for formal teaching.

There are no official instructions regarding textbooks. For certain parts of the syllabuses, the textbook is often replaced by notes made by the pupils themselves.

Teachers

Secondary teachers of mathematics are required to have completed five or six years of university study with mathematics as main subject and physics, chemistry and astronomy as auxiliary subjects, and a six-month course of teacher training, the theoretical part of which is given by the university, and the practical part of which takes place in a secondary school under the supervision of a competent person. The minimum required title is that of *candidatus magister ;* some teachers also hold a doctorate.

Some difficulty is at present experienced in recruiting secondary teachers of mathematics. When such a teacher is absent, it is not easy to replace him, and his colleagues among other mathematics teachers usually have to take over his lessons in addition to their own. The needs of secondary education have nevertheless been met.

Mathematics teachers in secondary and teacher-training schools have their own association. This association is actively concerned with the provision of further training for its members; it regularly organizes a conference lasting a few days in the autumn holidays, and a one-day meeting in spring; on one occasion it organized a seven-day conference in the long vacation, and now proposes to do so periodically, probably once every five years. The association also has reading circles which receive several foreign mathematics journals; in this way its members are able to keep abreast of the evolution of mathematics. Last year the University of Copenhagen organized a ten-week series of lectures taking up two hours a week, for secondary teachers of mathematics.

Mention should also be made of the important conference held in Aarhus last year, which was attended by over four hundred secondary teachers of mathematics and physics in Denmark, Finland, Sweden, Norway and Iceland.

Practically all secondary schools subscribe to the mathematical journal *Nordisk matematisk tidskrift*.

Miscellaneous

Legislation was recently passed, which authorizes secondary teachers of mathematics, subject to approval by a special committee, to depart somewhat from the ordinary lines of their work and undertake experimental teaching.

Modern developments in the mathematical field have certainly influenced the teaching of the subject at secondary level, especially in regard to textbooks, which have become more precise, and to syllabuses, in which the work on functions has been made more extensive and more advanced. It may also be said that the professional standard of the teachers is higher.

An institute of educational experimentation was recently established, and in due course will be in a position to publish the results of experiments on the teaching of mathematics.

DOMINICAN REPUBLIC

From the reply sent by the Secretariat of State for Education and the Fine Arts

Place of Mathematics in Curriculum

Secondary education follows six years of primary education, and two years of intermediate education. Its duration is four years, and the age range of the pupils from 15 to 18. Mathematics is taught as a compulsory subject in the 1st, the 2nd and the 3rd year, and in the physics and mathematics section of the 4th year.

Out of a total time-table of twenty-five hours a week, five hours are allotted to mathematics in the 1st and the 2nd year, four hours in the 3rd year, and twelve hours in the physics and mathematics section of the 4th year.

Mathematics is designated *arithmética razonada y álgebra* in the 1st year, *geometría plana* in the 2nd year, *geometría del espacio y trigonometría* in the 3rd year, and *elementos de matemáticas superiores, geometría y trigonometría*, and *dibujo lineal* (linear drawing) in the 4th year.

Mathematics ranks as a main subject in promotion examinations.

Aims of Mathematics Teaching

Mathematics is expected to make its contribution to the pupils' general education, and thus to their future activities, to prepare them for higher education, and to enable them to acquire certain mental habits favouring the development of personality.

Syllabuses

1st year. The fundamental operations, the nature of divisibility, the highest common factor and the lowest common multiple, fractions, square roots, proportional parts. Introduction to algebra, algebraical expressions and the corresponding operations, first-degree equations, systems of equations, equations of the second degree with one unknown, trinomials of the second degree, introduction to logarithms. — *2nd year.* Fundamental notions of geometry (straight lines, planes, angles), angles, polygons, notions of symmetry, the circle and the circumference, the calculation of areas, the theorem of Pythagoras, similar triangles and polygons, notions of projection. — *3rd year.* Geometry : solid geometry, the relative position of straight lines and planes, dihedral angles, orthogonal projections, polyhedrons and round bodies, the calculation of areas and volumes. Trigonometry : the measurement of angles, trigonometrical ratios, circular functions, the solution of triangles, elementary topography. — *4th year, physics and*

mathematics section. Higher forms of calculation, the laws of proportion, variables, logarithms and their applications, coordinates, functions and their graphical representation, Newton's formula, progressions, elementary calculation of probabilities. Higher geometry: trigonometrical ratios and the use of tables, elementary topography, problems.

The syllabuses also include the history of mathematics and the principles of axioms.

Methods of Teaching Mathematics

Official recommendations concerning the methods of teaching mathematics have been issued. Their main emphasis is on the necessity of preventing intuition, which is suitable for the adolescent stage, from being replaced entirely by the inductive and deductive reasoning commonly appealed to in the teaching of mathematics.

The teachers make use of all teaching materials which they deem necessary to make their teaching objective.

The syllabuses include some practical exercises, involving direct observation, constructions, measuring, etc.

Teachers

Secondary teachers of mathematics are required to hold the certificate of *Maestro normal de segunda enseñanza*, with mathematics as special subject, or a university qualification.

Some difficulty was experienced in the past in recruiting secondary teachers of mathematics, but it was overcome through the establishment of higher normal schools for the training of specialist teachers, with boarding accommodation, and the grant of scholarships covering all the pupils' expenses.

Teachers have various means of obtaining further training at their disposal, including teacher exchanges, lectures, special courses, seminars, study weeks, and specialist teachers' associations.

Miscellaneous

Changes in the secondary mathematics syllabuses are envisaged, as it is felt that the elements which experts and technicians have introduced into them should be eliminated, and only those elements retained which can be assimilated by adolescents, as shown by experiments undertaken with a view to organizing the whole of school work on activity lines.

ECUADOR

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is a compulsory subject throughout the six years of secondary education (pupils from 12 to 18 years of age). The weekly time allocation is five hours (15.62%) out of a total of thirty-two hours, in the 1st and the 2nd year, four hours (12.50%) out of thirty-two hours, in the 3rd and the 4th year, three hours (9.09%) out of thirty-three hours, in the 5th year, and three hours (11.53%) out of twenty-six hours, plus an additional three hours for pupils choosing mathematics as option, in the 6th year.

The subject is designated *aritmética y geometría* in the 1st and the 2nd year, and *álgebra y geometría* in the remaining years. Mathematics has the same importance as other subjects in the curriculum, except that it has added importance in the secondary school leaving examination, in the case of pupils who have chosen the subject as option.

Aims of Mathematics Teaching

The aims of teaching mathematics are given as follows in the official syllabuses : " The immediate aim is to impart the necessary ability to do the calculations that an educated person is normally called on to do, and the elementary knowledge of the physical and mathematical sciences demanded by modern conditions. One of the main educational aims of teaching mathematics is to form and develop the habit of logical thought as manifested in concise and exact speech."

Others aims given in the official syllabuses are to fill in such gaps, in the 1st and the 2nd year, as the pupils' previous schooling may have left, and to enable the pupils to grasp the relationships and properties, and on this basis the problems and mechanism, of mathematical operations.

Syllabuses

1st year. Arithmetic : the four fundamental operations, vulgar and decimal fractions. Geometry : the geometrical elements of solids ; angles and quadrilaterials. — *2nd year.* Arithmetic : compound numbers, ratio and proportion, rule of three, percentages, interest, proportional division. Geometry : triangles, area of regular polygons, circle. — *3rd year.* Algebra : algebrical numbers, addition and subtraction of monomials and polynomials, equality and inequality, products, division, common factors. Geometry : relationships between the parts of a triangle, parallel lines,

loci, relationships of the parts of the circle. — *4th year*. Algebra : algebraical fractions, equations, first-degree binomials, powers. Geometry : similar polygons, projections, Pythagorean theorem, elementary trigonometry. — *5th year*. Algebra : logarithms and their applications, arithmetical and geometrical progression. Geometry : solid geometry, parallelopipeds, round bodies. — *6th year*. Algebra : compound interest, annuities, permutation and combination, probabilities. Geometry : trigonometrical functions of a right-angled triangle, problems on right-angled and equilateral triangles. Optional work :— algebra : more advanced work on roots and fractional and negative exponentials, double radicals, elementary calculation of derivatives ; geometry : more advanced work in trigonometry, analytical geometry.

Methods of Teaching Mathematics

The basis of mathematical reasoning is the concept of " function ". It is to this concept, therefore, that pupils must be introduced, at first in a concrete and objective way, and then, in the upper classes, in relation to the generalization and graphical representation of individual cases.

EGYPT

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught at secondary level throughout the four years of the lower cycle. The age range of the pupils is from 10 to 14 years. Six hours a week (16.7% of the total time-table of thirty-six hours) is allotted to mathematics in the 1st and the 2nd year, and four hours (11.1% of the total time-table of thirty-six hours) in the 3rd and the 4th year.

In the upper cycle, where the age range of the pupils is from 14 to 17 years, mathematics is taken by all pupils in the 1st year (for four hours a week, representing 11.1% of the total time-table), and by the science section in the 2nd and the 3rd year (for seven hours a week, representing 18.4% of the total time-table). Pupils in the science section, moreover, who choose to do additional mathematics, do an additional three hours a week in the subject in the 2nd and the 3rd year.

Mathematics embraces arithmetic in all four years of the lower cycle, geometry throughout the lower and the upper cycle, algebra in the 3rd and the 4th year of the lower cycle and throughout the upper cycle, trigonometry and mechanics in the 2nd and the 3rd year of the upper cycle, and coordinate geometry in the 3rd year of the upper cycle.

Mathematics is invariably a required subject, except in respect of the additional mathematics already referred to. The syllabus for additional mathematics covers algebra, geometry, trigonometry, coordinate geometry, the history of mathematics, and elements of statistics.

Aims of Mathematics Teaching

Pupils in the lower cycle desire to solve their own problems and to understand their environment, and the mathematics syllabuses for this stage have been designed with this end in view. The arithmetic comprises such matters as fractions, measures, interest, and mensuration ; the study of algebra is elementary and practical ; the geometry taken is practical, and forms a good basis for further study.

In the upper cycle, as already noted, mathematics is taken in the 1st year by all pupils, and in the 2nd and the 3rd year only by pupils in the science section. The aim of mathematics teaching in the 1st year, therefore, is to give the pupils who will proceed to the science section sufficient knowledge to pursue their studies, and the pupils who will proceed to the literary section a sufficient mathematical background, and

to train all the pupils to acquire the scientific mode of thinking in problem solving.

Syllabuses

In the lower cycle, arithmetic is taught throughout the four years of the course, elements of practical geometry in the 1st, the 2nd, and the 3rd year, theoretical geometry in the 3rd and the 4th year (triangles, areas, etc.), and algebra in the 3rd year (symbols, easy equations) and the 4th year (simultaneous equations of the first degree).

In the upper cycle, algebra (factorization, equations, graphs) and geometry (loci, the circle, scale drawing) are taught in the 1st year. The required course for the science section in the 2nd year embraces algebra (quadratic equations, fractions, indices, logarithms), geometry (applications of the theorem of Pythagoras, similarity of triangles), trigonometry (numerical applications), and mechanics (kinematics, gravity, the parallelogram of forces, etc.). The additional course for the science section in the 2nd year comprises variation, similar figures, the history of mathematics, and elements of statistics, etc.

The required course of the science section in the 3rd year comprises algebra (progressions, permutations and combinations, approximation and error, etc.), trigonometry (circular measure, sector and segment, etc.), mechanics (elementary dynamics and statics), and coordinate geometry (the straight line). The additional course of the science section in the 3rd year embraces the theory of quadratic equations, solid geometry, trigonometrical relations, and coordinate geometry applied to the circle.

The history of mathematics is included in the additional course of the science section in the 2nd year of the upper cycle. Some reference is made to axioms in the syllabus of theoretical geometry.

Methods of Teaching Mathematics

The official instructions and suggestions concerning methods of teaching mathematics at secondary level may be summarized as follows.

In the 1st and the 2nd year of the lower cycle, in regard to arithmetic, the work should be based at first on visual aids and illustrations, oral and written work should be given in each lesson, so that the pupils may be trained to be quick and accurate in thinking, the pupils should be trained to perform with skill and understanding the different processes and to understand the data of the problems, and theses should be taken from real life (prices, statistics). The work in practical geometry should train the pupils to use the instruments properly, and the practical and scale drawing exercises should be taken from real life.

In the 3rd and the 4th year of the lower cycle, the pupils should be trained to express themselves accurately, and should learn the exact meaning of the mathematical terms used. In order to avoid the danger of pupils memorising and solving problems mechanically, however, terms and formulas should be used with restraint. In regard to both theoretical and practical methods of proof, the pupils should be trained to collect facts, arrange them, and find out the rule; the steps of proof should be

logically arranged. The social side of the topics taken and their connection with life should be taken into consideration ; pupils should know, for example, what actually happens in regard to compound interest and shares. The formulas ought to be those used in other sciences, or in respect of natural, social, or economic phenomena. Direct riders and numerical examples should follow each theorem. In teaching any geometrical fact, moreover, the teacher should draw, not merely one, but several figures, of types which the pupils are likely to come upon ; the grouping of theorems dealing with the same topic is also regarded as important. The pupils should be trained to use the analytic as well as the synthetic method.

In the 1st year of the upper cycle, in regard to both geometry and algebra, teachers should proceed from the general to the particular, and, with the help of numerical examples, from the particular to the general. The pupils, moreover, should be trained to draw graphs for different values, and to use them in solving equations of the second degree.

In the 2nd and the 3rd year of the upper cycle, ratios and proportions should be treated both algebraically and geometrically. The pupils should be trained to use logarithmic tables. It is considered advisable for a teacher to give the trigonometrical ratios of an angle one at a time, not all at once. The pupils should be trained to use a theodolite to measure heights and distances, and, if possible, taught to make a simple one at school. In mechanics, examples from real life should be given ; one of the main defects of teaching in this connection is the tendency to make the study simply a question of mechanical procedures and applications of formulas, without any real understanding of the ideas and facts ; the understanding, for instance, of velocity, acceleration, and force and its effects, is much more important than mere application of formulas. The apparatus, moreover, used for experiments in mechanics, may be defective, and the results obtained may thus be inaccurate ; in such a case, the pupils should be trained to adjust the apparatus, and to define the sources of error and correct for them.

The instructions concerning methods of teaching mathematics at the transfer stage from primary to secondary education have been indicated above. The correlation of mathematics with other subjects is attended to in the formulation of syllabuses and instructions on methods, and in the textbooks used ; the examples given in the course of mathematics teaching have a close bearing on the other subjects.

Teaching aids are used to a great extent in mathematics lessons. A great variety of models are used, for example, for the teaching of geometry, and the pupils are encouraged to construct models themselves. Various kinds of apparatus are used for the study of mechanics : inclined plane, Atwood's machine, etc. Use is also made of films.

The mathematics syllabuses include applied mathematics. The mechanics course, for example, covers the elements of statics and dynamics ; a number of topics in trigonometry and mechanics are dealt with in a practical way before the formal teaching.

The Ministry of Education controls the textbooks used for mathematics teaching. Some of these books are the results of competitions, others are chosen by commissions composed of experienced teachers, inspectors, etc. Non-controlled books are also used.

Teachers

Mathematics teachers for the upper cycle of secondary education are required to hold either a first degree in mathematics and a diploma (one-year course) of the Institute of Education, or the diploma of the Higher Training College (four-year course covering mathematics and educational theory and practice).

Mathematics teachers for the lower cycle of secondary education are required to hold the diploma of the Special Training Institute (three-year course, following the baccalaureate examination, covering mathematics, science, and educational theory and practice).

Great difficulty is at present experienced in providing the upper cycle of secondary education with mathematics teachers. This is due to the shortage of university graduates in mathematics. The Higher Training College, referred to above, and which is due to open in June, 1956, is expected to assist greatly in overcoming this difficulty.

Secondary teachers of mathematics keep up-to-date in their knowledge through lectures and seminars arranged during the school year and the summer vacations by the in-service training department. Scholarships are awarded for further training at home or abroad.

Miscellaneous

Modifications are contemplated in the teaching of mathematics in secondary schools. They comprise the more widespread use of audio-visual aids, the formation of mathematics clubs in the schools, bringing mathematics into closer touch with the problems of everyday life (the recent introduction of statistics served this purpose), and greater emphasis on the social and cultural aspects of the history of mathematics.

Experiments and investigations are now being carried out by the Institute of Education in Cairo, on the evaluation of the general mathematics course in secondary schools, the vocabulary of algebra and geometry, the teaching of informal logic and its influence on the teaching of geometry, and the construction of a general mathematics course.

FINLAND

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught in all years and sections of Finnish secondary schools.

The commonest type of secondary school has an eight-year course following the four-year primary course. The usual age of entry is eleven years.

The time allocated to mathematics in the eight-year boys' and mixed secondary schools is as follows :— first cycle (1st to 5th year) : five hours a week in the 1st year, three hours in the 2nd, and four hours in the 3rd, the 4th and the 5th ; second cycle (6th to 8th year) : five hours a week in the mathematics section, and three hours in the languages section, in all three years.

Mathematics is designated arithmetic in the 1st, the 2nd, and the 3rd year, algebra in the 2nd to the 8th year inclusive, geometry in the 3rd to the 7th year inclusive, and trigonometry in the 8th year.

Mathematics is a compulsory subject, but in the written part of the baccalaureate, a pupil may choose between the mathematics examination and the science and arts examination.

The same importance is attached to mathematics as to other subjects, in the evaluation of pupils' work.

Aims of Mathematics Teaching

The aims of teaching mathematics are to develop the pupils' capacity to think logically and work in accordance with a well defined plan, and to enable them to make accurate use of concepts, and present problems and the factors influencing the results clearly, concisely and grammatically. The subject is also expected to foster the pupils' ability to represent natural phenomena and objects and human activities in quantitative terms, to enable them to think functionally and to grasp the idea of space, and to show them the importance of mathematics in science, technology, economic affairs, and national defence. The subject, furthermore, should give pupils practice in calculating with the aid of finite and general numbers, and in solving quantitative problems relating to daily life and to science, and familiarize them with the principles of plane and solid geometry. In the upper classes mathematics should also be taught in such a way as to constitute a sound basis for the beginning of higher studies.

Syllabuses

1st year. Arithmetic : the number system, the decimal system, measures, diagrams; addition, subtraction, multiplication and division of whole numbers ; ratios, much practice in mental calculations. — *2nd year.* Arithmetic : areas and volumes ; measurement of time, of angles, of arcs, etc. ; divisibility of whole numbers ; vulgar fractions ; percentages, interest ; much practice in mental calculations. — *3rd year.* Arithmetic : ratios and comparisons, functions, linear diagrams, simple rule of three, simple proportional parts ; much practice in mental calculations. Geometry : completion of geometry learned in arithmetic lessons ; properties of angles, triangles and the circle. Algebra : laws of calculation with general and finite numbers ; simple first-degree equations ; mental work. — *4th year.* Algebra : elements of graphical representation ; ratio and proportion ; operations with polynomials ; simple first-degree equations. Geometry : properties of rectilinear figures and the circle ; simple problems of construction ; mental work. — *5th year.* Algebra and arithmetic : graphical representation of a whole function of the first degree with one variable ; first-degree equations in one unknown ; simple problems ; roots ; revision of arithmetic ; shares, debentures, discount ; tables for calculation ; mental work. Geometry : properties of the circle ; similarity of triangles ; trigonometrical functions of acute angles ; theorem of Pythagoras ; more advanced work on areas and volumes. — *Mathematics Section : 6th year.* Algebra : theory of indices and radicals ; irrational numbers, definition of imaginary and complex numbers ; graphical and algebraical method of solution of second-degree equations, graphical and algebraical solutions of first-degree equations in two unknowns, first-degree equations in three or more unknowns ; problems. Geometry : more advanced work on proportions ; theory of similarity ; solving geometrical problems with the help of algebra ; theory of the area of polygons. — *7th year.* Algebra : simple second-degree equations in two unknowns ; properties of roots of second-degree equations ; advanced graphical treatment of first-degree function ; derivation of the equations of the circle, the ellipse, the parabola and the hyperbola ; the derivative and its applications to the second-degree trinomials ; problems on maxima and minima ; logarithms ; arithmetical and geometrical progressions ; compound interest. Geometry : measuring the circle ; drawing figures of solid geometry ; solid geometry. — *8th year.* Algebra and geometry : revision. Trigonometry : solving right-angled triangles ; general definitions of trigonometrical functions ; goniometry ; solution of acute-angled triangles ; applications to geometry, physics and geography. *Languages Section : 6th year.* Algebra : short account of the theory of indices and roots ; irrational and imaginary numbers ; graphical and algebraical solution of second-degree equations ; simple problems ; sum and product of the roots of a second-degree equation. Geometry : brief revision of the theory of proportions ; similarity ; solving simple geometrical problems with the help of algebra. — *7th year.* Algebra : first-degree equations in two unknowns ; advanced study of first-degree functions by the graphical method ; logarithms. Geometry : area of polygons, measuring the circle ; introduction to solid geometry ; area and volume of simple solids. — *8th year.* Algebra and geometry : revision. Trigonometry : solution of right-angled triangles.

Methods of Teaching Mathematics

Suggestions to teachers concerning methods of teaching mathematics are officially issued, and may be summed up as follows. — *Arithmetic.* In the lower classes teachers should accompany their explanations with simple numerical examples, and problems to be solved by the pupils themselves, and should use materials giving plenty of concrete examples. They should also make use of geometrical figures, and thus prepare the pupils for algebra and geometry. The study of measures should go hand-in-hand with actual measurement. In all the years in which arithmetic is taught, emphasis should be laid on mental calculations. — *Algebra.* In the initial stages of the course, the subject-matter should be presented in as living and concrete a way as possible. Teachers should give special attention to the graphical representation of functions, and to the properties of functions. The use of the slide rule should be taught as an application of the theory of logarithms. — *Geometry.* Teachers should begin geometry by giving their pupils a clear idea, based on observation, of the first elements of the subject, and practice in the use of simple instruments. Even when the work becomes more systematic, teachers should continue to make their teaching living and concrete. Models, in many cases made by the pupils themselves, should be used in solid geometry. — *Trigonometry.* The main objective of teaching trigonometry should be to give the pupils a clear idea of the definitions of trigonometrical functions and variations of functions, and of their interdependence.

In all years special attention is paid to estimating the degree of accuracy of approximate numbers.

The methods proposed were designed to ensure a smooth transition from the mathematics teaching at primary level to that at lower secondary level.

Teachers endeavour at a fairly early stage to give their pupils the mathematical basis necessary for studying physics, and bear the application of mathematics to physics in mind, when setting exercises. They also seek to find applications of mathematics to social phenomena.

The materials used for mathematics lessons are ample, and include geometrical models. Films and filmstrips are coming into general use.

Teachers

Secondary teachers of mathematics are required to hold a first degree in philosophy, covering mathematics, physics and chemistry, and to have completed one year of professional training in a normal school, comprising practice teaching, lectures, criticism, and a final teaching test.

Secondary teachers of mathematics are classified either as "first class teachers" (who begin their career at lower secondary level) or "second class teachers" (whose work is mostly at upper secondary level). Each normal school has its professor of mathematics.

There has been a shortage of secondary teachers of mathematics for some years. An estimate made in 1955 shows that 500 more of them will be required in the next few years ; the actual annual increase in their

numbers, however, is only between 15 and 20. The main reasons for the shortage are probably the lowness of the salaries as compared with the length of the period of study (six years at a university and one year of professional training), and the high academic requirements.

In recent years the general directorate of schools, in collaboration with the University of Helsinki, has organized summer courses for persons wishing to qualify as mathematics teachers.

The Finnish association of mathematics and physics teachers organizes courses and study days each year, and publishes a mathematical journal called *Matemaattistan aineiden aikakauskirja*. Mathematics teachers from the Scandinavian countries organize an annual congress.

Since 1940 no travelling scholarships have been granted.

Miscellaneous

The teaching of mathematics will come up for careful consideration in connection with the reform of secondary education now in progress. It is proposed to rid the syllabuses of useless knowledge, and to bring them into closer conformity with the level of each year. Endeavours are also being made to extend the study of differential and integral calculus in the mathematics section.

FRANCE

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

Mathematics is taught throughout secondary education in France. The age range of the pupils is from 11 to 18.

The following table shows the number of hours allotted to mathematics per week, and the percentage of the full time-table devoted to mathematics:

	Mathematics Hours	%
1st Year:		
Classical section	2	8.33
Modern section	2	8.70
2nd Year:		
Classical section	2½	10.87
Modern section	2½	11.36
3rd Year:		
Classical section	2½	10.00
Modern section	2½	10.40
Modern short section	3	13.04
4th Year:		
Classical section	2½	9.84
Modern section	2½	10.00
Modern short section	3	12.50
5th Year:		
Classical section:		
(a) Latin-Greek	1½	4.74
(b) Latin-Greek-science	4	15.00
(c) Latin-2 modern languages	1½ +1 1/3 optional	6.20
(d) Latin-physical sciences	4	16.00
(e) Latin-physical and natural sciences	4	14.20
Modern section:		
(a) Language, physics	4	15.40
(b) 1 language, physical and natural sciences	4	16.00
Technical economic section	4	14.40

	Mathematics Hours	%
6th Year:		
Classical section:		
(a) Latin-Greek	1½	6.30
(b) Latin-Greek, science	4	15.20
(c) Latin-2 modern languages	1½ +1½ optional	6.30
(d) Latin-physical sciences	4	17.00
(e) Latin-physical and natural sciences	4	14.50
Modern section:		
(a) Languages, physical sciences	4	15.70
(b) 1 modern language, physical and natural sciences	4	17.00
Final Year:		
Philosophy	1½	6.40
Experimental science	4	15.70
Mathematics	9	33.30

Mathematics appears in the secondary curriculum as such.

The subject is a compulsory one in all classes, except that some classical sections have certain options for pupils intending to continue their studies at higher level.

In promotion examinations, which generally consist of one science paper and three arts papers, no subject has a higher mark than mathematics. A pupil who does well in mathematics may therefore pass, even if he does badly in other subjects.

In both parts of the baccalaureate, there is invariably at least one paper in mathematics. Certain classical sections have only one paper in mathematics, and pupils may choose whether to take it orally or in writing. In every case a pass in the baccalaureate is based on the pupil's general average, with the proviso that he fails if he gets a nought in any subject.

Aims of Mathematics Teaching

No specific aims are assigned to mathematics teaching at secondary level. French educationalists regard secondary education as an organic whole; they admit the necessity of drawing up a curriculum in terms of subjects, but avoid assigning specific aims to each subject, because it would involve a limitation of the purpose of secondary education. Instructions issued on 30th September, 1938, and a circular entitled *Les buts de l'enseignement du second degré* (the aims of secondary education), issued on 29th September, 1952, make a special point of giving only the aims of secondary education taken as a whole, and stress the need of forming an élite with developed capacities of thought, feeling and will, and a strong sense of morality and of beauty.

Syllabuses

1st year (classe de sixième). Numerical exercises based on the knowledge of measures of length, area, volume and capacity, weight, angles, etc. — *2nd year (classe de cinquième).* Arithmetic : operations with whole numbers and decimals. Geometry : straight lines, plane figures, angles, congruence of triangles, constructions. — *3rd year (classe de quatrième).* Arithmetic : resolution of whole numbers into prime factors ; determination of the highest common factor and the lowest common multiple. Algebra : introduction to algebraic calculations ; numerical first-degree equations in one unknown. Geometry : congruence of triangles, properties of polygons ; the circle, arcs, chords, etc. — *4th year (classe de troisième).* Arithmetic : ratio, proportion, square root. Algebra : polynomial with one variable ; first-degree equations and inequalities in one unknown. Geometry : loci, similar triangles ; elements of trigonometry ; area of polygons. In addition (short modern section) : solution of second-degree equations in one unknown ; solid geometry. — *5th year (classe de deuxième).* Algebra : function of a variable ; first-degree equations in two unknowns, second-degree equations involving one unknown with numerical coefficients. Geometry : ratios of two segments of a straight line ; similar triangles, elements of trigonometry ; sine, cosine and tangent of an acute angle. In addition (in " C " and modern classical sections) : discussion of first-degree equations and inequality involving one unknown, and of a system of two equations in two unknowns ; geometry : loci, the circle, ratios, power of a point with regard to a circle, perimeter of a circle. — *6th year (classe de première).* Algebra : the general equation of the second degree involving one unknown ; variation of a trinomial of the second-degree with numerical coefficients. Geometry : plane and straight lines ; orthogonal projection on a plane ; formulas of the volume of prisms, pyramids, cones and cylinders, and of their lateral areas ; area and volume of spheres. In addition (in " C " and modern classical sections) : definition and geometrical significance of the derivative of a function for a given value of the variable ; trigonometry : circular functions, equations ; geometry : use of a side projection to determine or represent a solid. — *Philosophy class.* Trigonometry and algebra : concepts necessary for studying the physics syllabus, including the use of elementary functions. Cosmography : the heavens and the constellations ; the earth, the sun, the solar system, the planets, the moon, the stars.

The official mathematics syllabuses make no explicit reference to the history of mathematics or to the principles of axioms. A circular issued on 6th October, 1952, however, stresses the necessity of giving some attention in the final year to the ways in which the various discoveries were made, of making use, in other words, of the historical method, which offers the only means of showing how creative thought, experimentation and systematic research have led up to the knowledge of today.

Aspects of the history of mathematics and of modern mathematical thought are also included in the syllabus of logic and the philosophy of science, taken in the final year.

Some teachers have organized their work on new lines by introducing the principles of axioms at a very early stage (about the age of 12).

An inquiry is now being made on this question, and will be the subject of a special number of the *Cahiers pédagogiques pour l'enseignement du second degré.*

Methods of Teaching Mathematics

Suggestions to teachers have been officially issued on teaching methods for secondary education in general, and also for mathematics in particular. The latter suggestions are essentially an adaptation to mathematics teaching of principles that are valid for all subjects. They deal with the aims of teaching mathematics at secondary level, the nature and spirit of the subject, methods (activity ones), and classroom organization and procedure. The recommendations made under the latter heading include the following : to begin the study of a given question only when the whole class can follow it ; clearly and honestly to indicate points which at a given level must be admitted without demonstration ; to avoid giving anticipatory " explanations ", under the pretext of " opening a window ", which are obviously above the pupils' heads ; to eliminate, except possibly in the form of exercises, all questions which are doubtless of great interest, but do not come within the syllabus ; to forego all demonstrations, even if deemed " beautiful ", which are far-fetched or too involved, graceful presentation being desirable only if it does not militate against clarity.

The suggestions also stress the need of instituting periods of directed work, as they appear to constitute the most suitable way of fostering the development of intellectual aptitude.

Care has been taken to ensure the continuity of mathematics syllabuses and teaching methods in the period of transition from primary to secondary education.

The mathematics syllabuses are to a certain extent integrated with those for physics, certain parts of geography, and handicrafts. The teachers of these subjects also hold special meetings to secure the greatest measure of integration possible. Endeavours are made, moreover, to integrate mathematics with other subjects such as French (precision of speech) and history (the history of scientific thought).

A number of films have been made for the teaching of mathematics at secondary level, geometry in particular. The materials used for mathematics lessons include surveying instruments (theodolite, chain, etc.), which the pupils have in some cases made themselves, together with special rules and cubes for the study of special products and various algebraic formulas. Materials for the construction of geometrical models are also available.

The secondary mathematics syllabuses now in force were not drawn up with any idea of direct practical application in mind. The intention was rather that of introducing the children to certain forms of thought and to teach them fundamental techniques, and as a result the place given to definitely practical application is limited. Teachers are always expected, however, to begin from the children's level of experience, and to get the children themselves to assist in building up the work through making explanations and applying knowledge already acquired to new questions. Teachers are recommended, moreover, as noted above, to institute periods of directed work.

Full advantage is taken, furthermore, of every opportunity of using mathematical knowledge in handicrafts lessons.

Exercises in surveying are frequently given, in connection with the study in geometry of the properties of right-angled and similar triangles, etc.

The official suggestions advise teachers to base their work on a textbook, which they are free to choose themselves. At the same time they remind teachers that it is not necessary to follow the textbook slavishly, whether in connection with matters of principle or of detail, provided that the children are always kept clearly informed of any changes made.

There should be no overlapping between textbook and notebook. In this connection the official suggestions to teachers state that it is beneficial if the pupils keep a written record of what they have learned in each lesson (definitions, postulates, figures, steps in a proof, etc.), but that such record will need to be most carefully supervised by the teacher. Teachers should never dictate their lessons, and should also see that no pupil takes notes continuously, in a vain endeavour to take down all that is said ; the disadvantages of such practices are obvious. At the same time, there is no reason why a teacher should not dictate a summary of work done, or a brief passage to amplify or modify some point in the textbook ; a short dictation of this kind represents a desirable activity if it takes the form of a rounding-off by teacher and class together of the questions dealt with.

Teachers

The training requirements of secondary teachers are set forth in a special pamphlet *(La formation du personnel enseignant secondaire)*. So far as their general training is concerned, secondary teachers of mathematics are required to hold at least a first degree in mathematics teaching *(licence d'enseignement de mathématiques)*, which comprises certificates of higher study in general mathematics, differential and integral calculus, rational mechanics, and general physics. In regard to professional training, they are required to hold the secondary teacher's certificate *(C.A.P.E.S.)*, which is granted to successful candidates of a competitive examination covering theory and practice. On passing the theoretical part of this examination, candidates are required to do practice teaching for one year, and are then eligible to sit for the practical part.

Persons who have passed the *agrégation* examination are also appointed as mathematics teachers. For admission to this examination candidates are required to hold an advanced diploma in mathematics or the equivalent, in addition to a first degree in mathematics teaching.

The preliminary tests for the *agrégation* examination differ according to the sex of the candidate, but the final tests are the same for both sexes.

Mathematics teachers are as difficult to recruit as other teachers required to hold advanced scientific qualifications. At the present time a person with scientific qualifications can easily find a post in industry which is better paid than teaching. The supply of persons with arts qualifications, on the other hand, exceeds the demand, and the solution of the double problem is probably only to be found in suitable counselling given in the course of secondary studies.

Secondary teachers of mathematics are grouped in a number of associations. All of them very active, especially the *Association des professeurs de mathématiques de l'enseignement publique*, which publishes a quarterly bulletin covering scientific and teaching matters. The *Cahiers pédagogiques pour l'enseignement du second degré* often publish studies on the teaching of mathematics, and the *Société française de pédagogie* organizes frequent discussions and lectures on the same subject. An important part in this field is also played by the regional branches of the associations and of the various " new education " groupings.

The most important of the journals published for secondary teachers of mathematics are the following: *Revue de mathématiques spéciales; Astronomie; L'Education mathématique; Journal des mathématiques élémentaires; Cahiers pédagogiques pour l'enseignement du second degré* (referred to above); *Les humanités scientifiques.*

Secondary teachers have opportunities of obtaining grants to participate in certain conferences and scholarships for further study abroad. They may also be seconded to the *Centre national de la recherche scientifique* (national scientific research centre) for the purpose of doing scientific or teaching research.

Miscellaneous

No large-scale reform of mathematics teaching appears to be likely in the near future, although the number of studies, experiments, and inquiries in this connection is increasing. The ideas under examination mainly relate to axioms, the calculation of probabilities, and the integration of mathematics with other subjects, physics in particular.

This report deals with mathematics only as far as the baccalaureate. Following the baccalaureate there are higher and special mathematics classes, which are the normal means of access to the leading science schools for the training of engineers, among others, and personnel for the larger national industries *(école polytechnique)*. The fact that these classes are preparing for extremely stiff examinations give them a rather special position, which has considerable influence on the syllabuses and methods of the secondary classes preceding them.

The syllabuses of these time-honoured classes have already had to undergo modification in the light of modern scientific and technological developments, and in time this will also lead to a new approach to the teaching of elementary mathematics.

GERMAN FEDERAL REPUBLIC

From the reply sent by the Conference of Ministers of Education

Place of Mathematics in Curriculum

The course of the grammar schools *(Gymnasien)* covers nine years of study, from the 5th to the 13th year of schooling inclusive. The age range of the pupils is from 11 to 19 years. Grammar schools are of three types: classical, scientific, and modern. Each of these types has three three-year cycles: the lower cycle (5th-7th years inclusive), the intermediate cycle (8th-10th years inclusive), and the upper cycle (11th-13th years inclusive).

Mathematics is taught in all years, except the 13th year of the modern grammar schools in Hamburg and of the modern and classical grammar schools in Lower Saxony (apart from two hours a week on mathematical problems) and Schleswig-Holstein.

The course of the general secondary schools *(Aufbauschulen)* begins in the 8th year of schooling, and that of the secondary schools in Bremen and Berlin in the 7th year of schooling. Mathematics is taught in these schools up to the 13th year of schooling.

The number of hours per week allocated to mathematics (out of a total time-table of 32 to 36 jours) is as follows:

	5th	6th	7th	8th	9th	10th	11th	12th	13th
Classical Schools	4-5	4-5	4	3-4	3-4	3	3	3	3
Modern Schools	4-5	4-5	4-5	3-4	3-4	3-4	3	3	3-4
Scientific Schools	4-5	4-5	4-5	4-5	4-5	4-5	3-5	3-5	3-5

Mathematics is designated *Mathematik-Unterricht* or, in the 5th to the 7th year in Rhineland Palatinate, *Rechenunterricht*.

Mathematics is compulsory in all years. In several *Länder* there are in addition optional study circles in the upper cycle of the grammar schools, and descriptive geometry is often taught as an optional subject.

The teaching of mathematics is considered as being of special value for training in the scientific outlook.

The four written papers for the secondary school leaving examinations must always include one in mathematics. In schools where mathematics is not taught in the 13th year, this paper in mathematics is taken at the end of the 12th year.

Aims of Mathematics Teaching

In all *Länder* the aims of mathematics teaching are to awaken and develop the pupils' intuitive faculties, to impart accuracy in thought and expression, to give understanding of the relation between premises and conclusion (axiomatic thinking) and of the system of relationships (functional thinking), to make the pupils aware of the beauty of mathematics, and to enable them to concentrate, make good models and drawings, and work methodically.

In all the curricula mathematics is treated as a separate subject. Mathematics is of considerable practical use, especially in work on physics and technics and in natural science lessons.

Syllabuses

1st and 2nd years (classes 5 and 6). Calculation with whole numbers, vulgar and decimal fractions, percentages ; elementary geometry, intuitive knowledge of parallelopipeds, rectangles, cubes and squares ; drawing figures with the aid of compasses, ruler and protractor ; calculations on national and foreign measures and currency ; and (in Bavaria) simple equations with one unknown. — *3rd, 4th, 5th and 6th years* (classes 7 to 10). Triangles, quadrilaterals, relative numbers, first-degree equations with one unknown, constructions and proofs ; first-degree equations with one or two unknowns, proportions, graphs, the theories of Euclid and Pythagoras, the circle, calculation of areas ; homology, congruence ; calculations involving the circle ; roots, second-degree equations, logarithms, the slide rule ; projection on one and two planes, calculation of volumes, trigonometrical functions, plane trigonometry. In some *Länder* the study of logarithms begins only in the 7th year ; in Hamburg vectorial algebra is begun in the 6th year. — *7th, 8th and 9th years* (classes 11 to 13). Exponential equations, arithmetical and geometrical progressions ; differential and integral calculus, with applications to problems in physics and geometry ; minimum and maximum, differentiation of products, quotients and roots, analytic geometry of conic sections, constitution of the whole of numbers.

In some *Länder* work is done in addition on partial integration and integration by substitution, the laws of series, the plane of Gauss, the theorem of Moivre, analytical transformation, and spherical geometry.

The tendency is to restrict the calculation of compound interest and of income to the study of series, and to replace spherical geometry by non-Euclidean concepts of spherical geometry.

Increasing emphasis is being given to the notions of displacement (translation, rotation, revolution of the plane of projection) in the intermediate cycle, and to the method of affinities and projections in the upper cycle. The vectorial representation of descriptive geometry is taught only in a few instances.

Practically all the syllabuses refer to the value of historical considerations, but without rigidly defining requirements in this connection. Increasing attention is being given to the principles of axioms.

Methods of Teaching Mathematics

Teachers are free to use what methods they prefer, provided they fulfil the aims of their subject. The most frequently used method is one where problems are prepared through discussions among the pupils themselves, the teacher intervening only when necessary. In the lower cycle priority is given to direct intuition and to mensuration. Teachers endeavour gradually to arouse the desire for accuracy in the work. The mathematics syllabuses of the 1st and the 2nd year (classes 5 and 6) continue the work done at primary level.

Correlation between mathematics and other subjects is ensured through frequent applications of the mathematical knowledge acquired to problems in physics, technics and, in some cases, economics. Mathematics and physics are often taught by the same teacher.

The materials used for the teaching of mathematics often include models of geometrical solids. Little use is made of the films produced by the educational films institute *(Institut für Film und Bild in Wissenschaft, Erziehung und Unterricht)* in Munich. The general tendency is to get the pupils to construct the necessary materials themselves.

The practical applications of mathematics are to be found mainly in connection with physics. In the intermediate cycle the pupils make drawings of plans and do cardboard work. The theodolite is used in the upper cycle.

Most of the *Länder* Ministries of Education publish a list of approved textbooks among which the schools may choose.

Teachers

Secondary teachers of mathematics are required to have completed eight semesters of university study in mathematics and one other subject (in science or arts), and two years at a teacher training school, and to have passed a preliminary examination (held during or at the end of their studies) in pedagogy and philosophy, and the State examination (held at the end of their studies) in mathematics and one other subject.

Such teachers, moreover, must pass the *Assessor* examination on concluding their two years of training and practice teaching at the teacher training school.

Such teachers are not required to hold purely academic degrees or examination certificates.

Secondary teachers, including those for mathematics, are in short supply in many *Länder*. University graduates are drawn into business firms by the offer of higher salaries than they would earn as teachers. No steps have as yet been taken to remedy the shortage of secondary teachers.

Most of the *Länder* have further training institutes whose courses include mathematics and the methodology of mathematics. Secondary teachers also get help in this connection through the bulletin published by the association for the development of the teaching of mathematics and natural science, and through the annual conference organized by this association. Many *Länder* make travelling allowances to teachers attending this conference.

Reforms, Miscellaneous

Reference has been made above to some of the present trends in the teaching of mathematics at secondary level. Experiments are being conducted in some schools on the theory of numbers and the theory of groups.

Each of the *Länder* has its own educational system. The teaching of mathematics is nevertheless to some extent uniform throughout the Republic, thanks to the collaboration of the *Länder* in the Conference of Ministers of Education, and to the coordination effected by the teachers' associations and annual conferences.

GREECE

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the six years of secondary education. The age range of the pupils is from 12 to 18 years. The hours per week allotted to mathematics are as follows: *1st year*, three hours (8.5% of the total time-table); *2nd year*, four hours (11%); *3rd year*, three hours (8.5%); *4th year*, four hours (10.5%); *5th and 6th years*, classical section, four hours (10%); *5th and 6th years*, science section, six hours (18%) and seven hours (20%) respectively.

Mathematics is designated practical arithmetic and geometry in the 1st and the 2nd year, theoretical algebra and geometry in the 3rd, the 4th and the 5th year, and theoretical trigonometry and geometry in the 6th year.

Mathematics is regarded as one of the main subjects in the secondary curriculum.

Aims of Mathematics Teaching

The specific aims of secondary mathematics are not officially formulated, but only those of secondary education as a whole. The teaching of mathematics is thus expected to make its contribution to the pupils' general education, to accustoming them to the discipline of scientific thought and of thinking for themselves, and to their preparation for their future tasks.

Syllabuses

Mathematics in the 1st and the 2nd year concentrates mainly on the practical aspects of arithmetic and geometry, and the deepening and widening of the knowledge acquired at primary level and the application of it to everyday problems.

In the succeeding four years, the subject proceeds gradually towards the generalization of calculation, that is to say, towards algebra and geometrical theorems, which provide the theoretical foundation of practical knowledge. In the upper years, trigonometry leads on to a study of cosmography, an introduction to astronomy.

The mathematics syllabuses do not include the history of mathematics, but the textbooks make some general references to it.

Methods of Teaching Mathematics

No official instructions or suggestions have been issued with regard to the methodology of mathematics teaching. The teachers of the subject

generally endeavour to apply modern educational ideas, in particular the principle of individual activity, and obtain good results in their work. No official reference has been made to the question of facilitating the transition from primary to secondary education.

Special care is taken to ensure coordination between the teaching of mathematics and that of experimental physics ; for the study of the latter subject, certain mathematical concepts are essential. The teaching of mathematics is also coordinated with that of geography, biology, and economics.

The teaching materials used for mathematics lessons include geometrical figures and solids, in the lower years, and scientific instruments, for the course on astronomy.

Very few application exercises are given, except for some elementary exercises on measuring (and the application of mathematical laws to accountancy in the schools of commerce). The geometrical drawing taken in the 5th and the 6th year of the science section includes architectural drawing and the drawing of simple machines.

There are definite official instructions with regard to textbooks. Textbooks are selected competitively ; authors are required to submit their books to the higher educational council, which chooses three of them for award, and has one of these three published by the official school publications service and made compulsory. The pupils also use non-official textbooks, collections of exercises in particular.

Teachers

Secondary teachers of mathematics are required to hold a first degree or a doctorate awarded by the mathematics department of the faculty of physical and mathematical sciences of a Greek university.

Some difficulty is being experienced at the present time in recruiting specialist secondary teachers. This is due partly to the lowness of the salaries, and partly to the attraction of life in the larger towns. In order to remedy the position, the authorities recently took steps to improve teachers' salaries and to facilitate their promotion.

After three years' service, secondary teachers of mathematics are given the opportunity of following further training courses at the teacher training school. These courses last two years, and supplement the teachers' academic training by familiarizing them with modern developments in the mathematical field, and give them instruction in pedagogy and teaching methods.

Secondary teachers of mathematics also have opportunities of securing further training through the regional conferences periodically organized by the inspectors of the subject. Most of the mathematics teachers, moreover, also belong to the Greek mathematics society, which publishes two journals, one on elementary mathematics, the other on pure mathematics. Articles on the teaching of mathematics are published at frequent intervals in the educational journals. Mention should also be made of the scholarships granted by the Ministry of Education to selected secondary teachers for further training abroad.

Miscellaneous

The higher educational council examines all school syllabuses, and makes recommendations with regard to any necessary modifications. The reform of secondary education is now under consideration, and will include a revision of the mathematics syllabuses.

As to the impact of modern developments in the mathematical field on mathematics teaching in secondary schools, it is to be noted that the competitive selection of textbooks, referred to above, is deliberately made at relatively frequent intervals, in order that the textbooks may be kept up-to-date, thus reflecting the evolution of this science.

Experimental research on the teaching of mathematics is being carried out at the teacher training school, and at the classical secondary schools, conducted on model and experimental lines, attached to the Universities of Athens and Salonika.

GUATEMALA

From the reply sent by the Ministry of Public Education

Place of Mathematics in Curriculum

Secondary education comprises a three-year lower cycle of general education, and a two-year upper cycle of preparation for the baccalaureate examination or primary teaching.

Pupils enter a secondary school at 12 to 13 years of age. Mathematics is taught as a compulsory subject in the 1st, the 2nd, the 3rd and the 4th year ; five forty-minute lessons a week are allotted to the subject, a greater amount of time than is allotted to any other subject in the secondary curriculum. Under the designation of *matemáticas*, the subject comprises arithmetic, algebra, geometry and, in the 4th year, trigonometry. In the 5th year, mathematics is replaced by cosmography, geology, and physical geography.

As already noted, mathematics has a greater time allocation than any other subject, but it ranks equal with other subjects in promotion and school leaving examinations.

Aims of Mathematics Teaching

References to the aims of mathematics teaching in the lower secondary years may be found in the official syllabuses. Those aims are to enable the pupils to understand the quantitative aspects of human activities (production, consumption, labour, etc.) and of space and time, and to give them mental discipline (accuracy, confidence, speed) and a grasp of facts in the economic and practical fields.

Syllabuses

1st year. Fundamental notions, numeration, the operations, the nature of divisibility, fractions and operations on fractions, the decimal and other systems of measurements, the operations on compound numbers, powers and roots. Geometry : the principal geometrical figures. — *2nd year.* Proportional parts, ratios and proportions, the rule of three and its applications, algebraical numbers and expressions, common factors, algebraical fractions, first-degree equations and the graphical representation of equations, powers and roots of algebraical expressions, second-degree equations. — *3rd year.* Logarithms and their applications, arithmetical and geometrical progressions, exponential equations. Geometry : properties of straight lines, angles and triangles, and a study of the theorems involved ; parallelograms, circles, and circumferences (theorems) ; dimensional rela-

tions in the elements of the triangle, areas (theorems). — *4th year.* Fundamental notions of trigonometry, the solution of triangles, trigonometrical formulas, fundamental notions of solid geometry.

The syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

No special methods are prescribed for the teaching of mathematics. Teachers are free to adopt the methods they deem the most suitable for achieving the desired ends, and it is therefore difficult to say which method is most widely employed. Nevertheless, the methods employed include the method of proceeding deductively from the theoretical concept to the practical application, that of proceeding inductively from the concrete to the theoretical, and that of combined induction and deduction.

Teachers are also free to choose what ways they deem best for facilitating the transition from primary to secondary education. In one large secondary school, for example, the pupils on entry are given a test to determine the level of their knowledge and the way they reason, and then, on the basis of the test, are formed into three groups which are each given appropriate teaching with a view to filling in gaps and to forming a homogeneous class.

Teachers take every care to ensure the coordination of mathematics with such subjects as physics, chemistry, and economics, mainly by choosing exercises and problems connected with these subjects (specific gravity, pressure, resistance, interest, foreign exchange, stocks and shares, etc.). They endeavour, moreover, to show the practical applications of the knowledge acquired, by choosing problems connected with everyday life ; they also give some exercises on construction and measurement, with a view both to such applications and to preparation for the introduction of theoretical concepts.

Teachers

The faculty of arts of the University of San Carlos has a special section for primary teachers wishing to train for the teaching of mathematics at secondary level. The qualification secondary teachers of mathematics are required to hold is thus awarded by this faculty. The academic training in mathematics proper is provided by the school of engineers [1]. The qualification in question, however, was only recently instituted, and the prescription that candidates for secondary mathematics posts must hold it, is therefore not absolute, preference nevertheless being given to those who do. Experienced teachers not holding the specialist diploma were formerly awarded a diploma by the Ministry of Public Education.

Every endeavour is made to provide further training for secondary teachers through the organization of lectures, conferences, and seminars, the grant of scholarships, etc.

[1] See " Secondary Teacher Training " (Paris, Unesco ; Geneva, International Bureau of Education ; 1954), p. 92.

HAITI

From the reply sent by the Department of National Education

Place of Mathematics in Curriculum

Mathematics is taught throughout the first six years of secondary education in all three sections, i.e., section A (Latin-Greek), section B (Latin-science), and section C (science-languages). In the 7th, the final, year, the so-called " philosophy class ", mathematics is taught only in section C.

The subject is always a compulsory one. The hours per week allocated to it are as follows :

	1st year	2nd year	3rd year	4th year	5th year	6th year	Phil. Cl.
Section A . .	5	5	4	3	3	4	—
Section B . .	5	5	6	5	5	5	— [1]
Section C . .	5	5	6	5	5	5	6

Mathematics is designated algebra (throughout the course), geometry (in and after the 2nd year), arithmetic (in the 1st, the 2nd, and the 3rd year, and in sections B and C in the 4th year), trigonometry (in sections B and C in the 5th year), projective geometry (in sections B and C in the 6th year), and algebra, further geometry, trigonometry, descriptive geometry, theoretical arithmetic, and cosmography (in the philosophy class).

Greater emphasis is placed on mathematics in sections B and C than in section A. In promotion examinations the mark obtained for mathematics is multiplied by two in section B and by three in section C. In the baccalaureate examination the mark is multiplied by three in section C.

Aims of Mathematics Teaching

The aims of mathematics teaching at secondary level are to complete the pupils' general education, to accustom them to the discipline of reasoning and thus to develop their judgment, to impart to them the mathematical concepts necessary for the study of other scientific subjects, and to prepare those with special aptitude for higher studies and a scientific career.

Syllabuses

1st year. Arithmetic : properties of numbers, highest common factor and lowest common multiple, fractions, operations on fractions. Algebra : algebraical numbers, algebraical expressions, the corresponding opera-

[1] There is no " philosophy class " in section B.

tions. — *2nd year.* Arithmetic : revision of the previous year's work ; the metric system, the rule of three, alloys and mixtures. Algebra : revision of the previous year's work ; algebraical fractions and the corresponding operations, first-degree equations. Geometry : straight lines, planes, angles, triangles (cases of congruence), polygons, circles. — *3rd year.* Arithmetic : general revision ; ratios and proportions, commercial arithmetic (interest, discount, shares, income), square root. Algebra : revision of the work done in the 1st and the 2nd year ; plus (in sections B and C) a more advanced study of first-degree equations (inequalities, irrational equations, the interpretation of negative solutions). Geometry : revision of the work taken in the 1st and the 2nd year. — *4th year, section A.* Algebra : revision ; equations of the first degree with one unknown, and the corresponding problems. Geometry : proportional lengths, similarity of triangles, dimensional relationships among the elements of the inscribed triangle and quadrilateral, regular polygons, areas, the circle and the circumference. — *4th year, sections B and C.* Theoretical arithmetic : decimal numeration, divisibility, prime factors, proportions. Algebra : revision ; a more advanced study of the equation of the second degree (coefficients and roots, biquadratic equations and inequalities, irrational equations, problems). Geometry : as for section A, in a more developed form, together with loci, homology, similar and inscribed polygons. — *5th year, section A.* Algebra : equations of the first degree with several unknowns, irrational equations. Geometry : revision of the previous year's work ; loci, constructions. — *5th year, sections B and C.* Algebra : revision ; variations and graphical representation of linear functions, parabolic and homographic functions, arithmetical and geometrical progressions, logarithms, compound interest, annuities. Geometry : revision ; relative position of straight lines and planes, cases of congruence of trihedrals. Trigonometry : circular functions, projections, the use of logarithmic tables. — *6th year, section A.* Second-degree equations, the graphical representation of functions, progressions, logarithms, compound interest, notions of derivates. Geometry : figures in space, the determination of a plane, the relative position of straight lines and planes, projection, symmetry, polyhedrons, round bodies, volumes and areas, sections, poles, tangent planes. — *6th year, sections B and C.* Algebra : derivatives and their applications to the study of variations of functions. Geometry : revision ; polyhedrons, the truncated prisms and pyramids, similar prisms and pyramids, areas of cylinders and cones, tangents and plane tangents, zone, spherical shell, sector, segment and spherical ring. Projective geometry : projections, revolution of the plane of projection, various problems. Trigonometry : revision of the previous year's work ; transformations, trigonometrical equations, the solution of triangles. — *Philosophy class.* Algebra : revision of the previous year's work ; trigonometrical functions, the evaluation of areas. Geometry : the transformation of figures, homology and similarity, stereographic projection, conic sections, developments. Descriptive geometry : the point, the straight line, the plane ; revolution of the plane of projection, change of plane, angles, plane section of polyhedrons. Trigonometry : revision of the previous year's work ; limits, trigonometrical functions. Theoretical arithmetic : numeration, divisibility, prime numbers, ratios, fractions, square and square root, absolute and relative error.

The history of mathematics is not included in the present syllabuses, but will be in the syllabuses being drawn up.

Methods of Teaching Mathematics

There are no official instructions concerning the methods of teaching mathematics at secondary level. Teachers generally give an exposition of the topic in hand, explain certain points, reply to questions from pupils, and set exercises of graduated difficulty.

No special measures have so far been taken to facilitate the transition from primary to secondary education, but this question is being studied in connection with the envisaged reform of syllabuses. The teaching of mathematics is correlated with that of other subjects, physics in particular, the mathematics and physics syllabuses being so arranged that the mathematical knowledge necessary for the understanding of given aspects of physics is taught in advance of those aspects.

The materials used for mathematics lessons comprise the usual instruments (ruler, compasses, set-square, protractor). When the new syllabuses are brought into force, films and filmstrips will also be used.

There are no official instructions regarding the textbooks used for mathematics teaching.

Teachers

Secondary teachers of mathematics are required to possess the secondary teacher's diploma awarded by the higher normal school on conclusion of a three-year course. Persons holding the baccalaureate (part two) certificate of section C (science) may be appointed as supply teachers.

Some difficulty is experienced in recruiting secondary teachers of mathematics, on account of the fact that the salaries offered are rather low. The education authorities are making every endeavour to raise secondary teachers' salaries, to the fullest extent allowed by the country's economic position.

The directorate of national education organizes conferences of secondary teachers of mathematics. Some scholarships are available for further training.

Miscellaneous

As noted above, new mathematics syllabuses are now being drawn up. The envisaged changes relate to the correlation between primary and secondary education, the methodology of mathematics, the practical applications in certain classes, and teaching materials (films, filmstrips, etc.).

HONDURAS

From the reply sent by the Secretariat of State for Public Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the five-year general secondary school course [1] (pupils of 13 to 17 years of age). The time allotted to mathematics is five hours (out of a total time-table of thirty-six hours) a week in the 1st and the 2nd year, and four hours (out of a total time-table of thirty-four, forty-three, and forty hours respectively) in the 3rd, the 4th, and the 5th year. The subject is designated *aritmética práctica* in the 1st and the 2nd year, *álgebra y geometría* in the 3rd and the 4th year, and *trigonometría* in the 5th year.

The same importance is attached to mathematics as to the other subjects in the secondary curriculum.

Aims of Mathematics Teaching

The aims assigned to the teaching of mathematics at secondary level are (a) to give pupils the knowledge and skill necessary for the solution of the mathematical problems of everyday life, (b) to develop their power of logical reasoning, and (c) to make them aware of the social significance of mathematics.

Syllabuses

1st year. Numeration, fundamental operations, division, highest common factor and lowest common multiple, decimal system, the monetary systems of Honduras and other countries of Central America. — *2nd year.* Revision of 1st-year syllabus, measurement of time, measurement of angles and problems on latitude and longitude, complex numbers, areas and volumes, powers and roots, proportional division, mixtures, exchange, bookkeeping. — *3rd year.* Introduction to algebra, algebraical expressions, operations, algebraical fractions, first-degree equations with one unknown. Geometry : geometrical elements, straight lines, surfaces, angles. — *4th year.* Fractional equations, second-degree equations, second-degree trinomial, functions and their graphical representation, Newton's binomial theorem, arithmetical and geometrical progressions, logarithms. Geometry : polygons, the circle, similar figures, polyhedrons, areas and volumes. — *5th year.* Introduction to trigonometry, triangles and trigonometrical functions, relations between trigonometrical functions, solution of triangles.

[1] The present inquiry does not cover teacher-training or vocational secondary schools.

Methods of Teaching Mathematics

The official mathematics syllabuses contain suggestions on the methods of teaching the subject. These suggestions do not stipulate the use of any one particular method, but deal in general with the intuitive approach, group work, direct observation, the use of concrete examples, etc. They make no reference to methods of ensuring a smooth transition from the primary to the secondary level.

The materials most commonly used for the teaching of mathematics include various objects, drawings, coins, and measuring instruments. Increasing use is being made at secondary level of organized visits to banks and to commercial and other productive undertakings, and a great number of exercises are given.

The secondary education directorate recommends what it considers to be the best textbooks.

Teachers

At secondary level mathematics is generally taught by engineers, commercial experts, and capable and experienced primary teachers. It is evident that they lack professional training, and, in order to remedy this, the education authorities intend to establish a higher school for training these and other secondary specialist teachers.

Secondary teachers of mathematics can obtain further training through special courses, vacation courses organized by the secondary education directorate, and the mathematics teachers' meetings held in the various secondary schools. Some of them are sent abroad for further study.

Miscellaneous

No changes are at present envisaged in the mathematics syllabuses. The subject has already been influenced considerably by modern ideas, and made more practical and " active " than was formerly the case.

HUNGARY

From the reply sent by the Ministry of Foreign Affairs

Place of Mathematics in Curriculum

Mathematics, under that name, is taught as a compulsory subject throughout the four-year course of general secondary schools.[1] The age range of the pupils is from 14 to 18 years. The hours per week and the percentage of the total time-table allotted to the subject are as follows :

	1st Year		2nd Year		3rd Year		4th Year	
	Hours	%	Hours	%	Hours	%	Hours	%
Classical section . .	5	16.1	4	12.5	4	12.1	4	12.5
Modern section . . .	5	16.1	6	18.8	5	15.2	4	12.5

Mathematics and the mother tongue are regarded as the two most important subjects in the secondary curriculum. Mathematics is included in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The aims of mathematics teaching at general secondary level are to teach the pupils to understand and formulate the laws governing their environment, to recognize the working of these laws in everyday life and nature, to represent spatial relationships graphically, and to acquire a systematic knowledge of geometrical figures and relationships, and of the concepts necessary for the solution of mathematical problems and the study of new problems. Mathematics teaching is also expected to develop the pupils' capacity for clear and systematic reasoning.

Syllabuses

Modern Section. 1st year. Transformation of algebraical expressions, equations of the first degree with one unknown, systems of equations with several unknowns. Geometry : triangles, quadrilaterals, polygons, circles, loci. — *2nd year.* Extraction of square roots, derivates, equations and systems of equations of the second degree, biquadratic equations, generalization of the concept of power and of logarithms. Geometry : cases of similarity, transformation of areas, the theorem of Pythagoras, surveying, trigonometry (the right-angled triangle). — *3rd year.* Arithmetical and geometrical progressions, trigonometry, the trigonometrical solution of triangles. Analytical geometry : the point, the straight line,

[1] The present inquiry does not cover the industrial and agricultural secondary schools, or the teacher-training schools at secondary level, also referred to in the reply from the Ministry.

the circle, the parabola, the ellipse, the hyperbola. — *4th year.* Functions, introduction to analysis, whole and fractional roots of equations of the highest degree, division of polynomials. Solid geometry : the cube, the prism, the cylinder, the pyramid, the cone, the sphere.

Classical Section. Essentially the same as for the modern section, but less advanced.

The official syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

Suggestions to teachers concerning the methods of teaching mathematics have been officially issued in the special methodology pamphlet *(Módszertani levél)* and in the handbook *(Útmutató)* published on the occasion of the annual technical consultative conferences. Instructions concerning the use of new textbooks are also issued to teachers.

The suggestions to teachers are mainly based on the desirability of combatting formalism, of coordinating theory with practice, and of the active participation of pupils in the lesson.

Steps are taken to facilitate the transition from the primary to the secondary stage. Coordination of mathematics with physics is ensured through the way in which the syllabuses have been drawn up, and through the specialization of mathematics teachers in both branches of study.

Various materials and aids are used for the teaching of mathematics, including films, filmstrips, models, wall charts, and other visual aids.

Importance is attached to practical exercises (surveying, the calculation of distances in the open air with the aid of trigonometry), inasmuch as one of the fundamental principles underlying the teaching methods used is to begin always with a practical problem and to use theoretical notions for the solution of practical questions. The importance given to practical activities, moreover, is increasing, on account of the introduction of polytechnical education.

As already noted, official instructions are issued concerning the use of textbooks.

Teachers

Secondary teachers of mathematics are trained through a four-year university course. The two main subjects which such a course always covers, are mathematics and physics. The teachers are also given training in pedagogy and teaching methods, and two one-month periods of practice teaching. The certificate they are awarded entitles them to teach mathematics in a secondary school. No difficulty is experienced in recruiting secondary teachers of mathematics.

Ample provision is made for the further training of secondary teachers, through the courses and lectures organized by the further training institutes, the lectures given under the auspices of the János Bólyai Mathematics Society, and the availability at the Socialist Education Library of books and specialist journals such as " The Mathematics Pamphlets " and " The Teaching of Mathematics ".

Miscellaneous

The envisaged combination of the classical and the modern sections in general secondary schools will affect mathematics syllabuses.

The existing mathematics syllabuses have certainly been influenced by modern developments in the mathematical field. The functional aspects of algebra are of increasing importance, and the notion of number and the study of functions is influenced by the theory of groups. Increasing emphasis is being given to geometrical transformations in the study of solid geometry.

Experiments are now in progress in certain general secondary schools, with the object of elaborating a uniform system of evaluation of mathematical work. Teachers engaged on research work of this kind reecive a monthly allowance from the Hungarian Academy of Sciences.

A considerable contribution to raising the level of pupils' knowledge is made by the competitions held at regular intervals, as announced in the " Mathematics Leaflets for Secondary Schools ". Such competitions are also organized at class level, to an increasing extent.

Mention should also be made of the booklets issued by the pupils' societies and study circles. Those issued in this way for mathematics cover, for example, the notions of the theory of numbers, and the calculation of probabilities, and further booklets are to be issued shortly on infinitesimal calculus and conic sections.

INDIA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Secondary education generally comprises a three-year "Middle School" stage (the 6th to the 8th year of schooling, pupils of 11 to 14 years old) and a two-year or three-year "High School" stage (the 9th to the 10th or 11th year of schooling, pupils of 14 to 16 or 17 years old).

Mathematics is taught throughout both these stages. It is designated arithmetic, algebra and geometry, but in some states also includes elementary mechanics and trigonometry in the upper classes. It is compulsory for boys in all years; for girls, arithmetic is compulsory, but algebra, geometry, etc., are optional.

Six forty-minute periods a week are allocated to mathematics (one seventh of the total time-table) at the "Middle School" stage, and eight or nine forty-minute periods (one fifth of the total time-table) at the "High School" stage.

All the subjects in the secondary curriculum are considered as being of equal importance.

Aims of Mathematics Teaching

The aims of mathematics teaching at secondary level are (a) to give the pupils the ability to understand, grasp and tackle with accuracy and confidence the problems of arithmetic connected with everyday life, (b) to develop the pupils' intelligence and give them training in abstraction, judgment and reasoning, and (c) to impart knowledge and ability to continue the study of the subject at higher level.

Syllabuses

There are no uniform syllabuses for the various states, but the principal questions treated are as follows:

Arithmetic: notation, numeration, the four fundamental operations, factors, highest common factor and lowest common multiple, fractions, square root, ratio and proportion, problems on proportional parts, percentages, simple and compound interest, mixtures, taxes, insurance, discount, work and wages, averages, calculation of areas and volumes.

Algebra: simple equations with one or more unknowns, factors, highest common factor and lowest common multiple, fractions, indices, surds, simple formulas and eliminations, simple graphs and equations of the first and the second degree.

Geometry: geometrical concepts and elements; theorems on angles, triangles and rectilinear figures, loci, the circle, similar triangles; constructions (inscribed and escribed polygons, circles, tangents, etc.).

Such elementary mechanics as is taught includes a study of simple machines (including wedge and screw), the lever, and measuring, etc. Trigonometry, where taught, involves simple trigonometrical ratios and relationships.

The secondary mathematics syllabuses do not include either the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

Official instructions have been issued concerning the methods of teaching mathematics. Teachers are recommended as far as possible to introduce the various topics through concrete problems with which the children are familiar, and to use simple, well-graded, concrete examples.

Detailed instructions, moreover, have been issued for the correlation of mathematics with other subjects, including physics, economics, geography, and social studies. Emphasis is also laid on correlation for the guidance of textbook writers. In practice, however, the teaching often leaves room for improvement in this connection.

The materials generally used for mathematics lessons include charts, models, and apparatus, and, wherever available, slides and films.

In most of the states the mathematics syllabuses do not include applied mathematics; in some states elementary mechanics is taught, as noted above. Activity methods are rarely used.

Official instructions exist concerning the choice and use of mathematics textbooks. These are generally selected by special committees appointed by the state governments for that purpose; they must adhere strictly to the prescribed syllabuses, their treatment of the subject-matter must be suited to the age-level of the pupils for whom they are intended, and they must satisfy the aims and objectives laid down for the teaching of the subject. A large number of mathematics textbooks have been published in the major Indian languages in the last few years.

Teachers

A secondary teacher of mathematics is generally required to possess a degree in the subject, and a degree or diploma in education or teaching. Untrained teachers are employed in some cases, where trained teachers are not available.

As a general rule no difficulty is experienced in recruiting men for the teaching of mathematics. Women teachers, however, who have specialized in mathematics, are not obtained so easily; where necessary, they are replaced by suitable men teachers.

Secondary teachers mostly lack facilities for further training. Refresher courses are organized by some state governments. A number of regional seminars were recently held in various parts of the country, at which secondary headmasters had the opportunity of discussing the

problems of secondary education, including teaching methods, and to exchange ideas. Another recent measure was the introduction of extension services in twenty-four training colleges, to help teachers to solve their problems and to improve their methods of teaching their various subjects, including mathematics.

Miscellaneous

The inclusion of elementary mathematics and statistics under " general science " is contemplated. General science would be taught during the first two years of secondary education, as a core subject. More detailed and advanced study of mathematics would, however, be provided, as an optional subject in the scheme of diversified courses.

Modern developments in the mathematical field have made the teaching of the subject at secondary level richer and more interesting.

Students preparing for the M. Ed. or Ph. D. degree sometimes take up the teaching of mathematics as a topic for advanced research. In one training institution, for example, tests are being constructed to diagnose pupils' difficulties in learning mathematics, and remedial measures are being sought.

The " Secondary Education Commission " has made certain bold and far-sighted recommendations concerning the reconstruction of secondary education. These recommendations include the use of dynamic methods in teaching, and the contemplated changes noted above in the secondary mathematics syllabuses. New secondary syllabuses are now being drawn up by the " Central Coordination Committee ", and will be circulated to state governments for adoption with such modification as they may consider necessary. As an optional subject mathematics will attain a much higher standard.

IRAQ

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the five years of secondary education, in all sections. The age range of the pupils is from 12 to 19 years. Seven hours a week (21.2% of the total time-table) is allotted to the subject in the 1st year, six hours (18.1%) in the 2nd and the 3rd year, and five hours (15.6%) in the 4th and the 5th year.

Mathematics appears in the syllabuses as " arithmetic and geometry " in the 1st year, to which is added " algebra " in the 2nd year onwards, and " trigonometry " in the 4th year onwards.

The same importance is attached to mathematics as to other subjects. A pass in mathematics is required in the secondary school leaving examination.

Aims of Mathematics Teaching

The aims of mathematics teaching may be summarized as being to accustom the pupils to thinking logically, and to solving problems with accuracy and speed.

Syllabuses

1st year. The fundamental operations, highest common factor, lowest common multiple, fractions, symbols, simple equations, involution, evolution, reduction, areas of polygons and solid figures, graphical representation of functions, ratio, percentage, the first thirty-five theorems of plane Euclidean geometry. — *2nd year.* Proportion, measures, proportional division, simple and compound interest, positive and negative quantities, simultaneous equations, brackets, thirty more theorems of plane Euclidean geometry. — *3rd year.* Factorization, the difference of two squares, factorizing the general trinomial, the sum or difference of two cubes, algebraic fractions, fractional equations, quadratic equations, graphs of quadratic equations in two variables, the remaining theorems of plane Euclidean geometry concerning the areas. — *4th year.* Exponents, linear functions, systems of quadratic equations, ratio, proportion, variation, theory of equations, logarithms, trigonometric ratios, simple trigonometric equations, solution of right-angled triangles, logarithms of trigonometric ratios, laws of sine, cosine and tangent, solution of triangles. — *5th year.* Simultaneous equations in two unknowns, progression, compound interest, annuity, the binomial theorem, loci of straight lines and planes in space, solids and theorems concerning them, study of the sphere, its definition, its elements, tangents and the theorems concerning them.

The syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

There are no official instructions concerning methods of teaching mathematics. Teachers commonly use the problem method. The education authorities, moreover, have not issued any special instructions in regard to ways of coordinating primary with secondary education. The necessary correlation between mathematics and other subjects in the secondary curriculum, however, has been ensured.

The materials used for mathematics lessons include drawings, slides, and geometrical solids. The textbooks are in most cases prepared by the Ministry of Education.

Teachers

Secondary teachers of mathematics are required to hold a first degree (B.A.) in mathematics awarded by the higher teacher-training school or other higher establishment. Those from the higher teacher-training school have received instruction in the special methods and techniques of teaching mathematics.

The Ministry of Education publishes a journal intended to help teachers to keep up-to-date in their knowledge.

Miscellaneous

Changes in the teaching of mathematics at secondary level are now under consideration ; one of the main problems being examined is the effect that modern developments in the mathematical field should have in this connection.

IRELAND

From the reply sent by the Department of Education

Place of Mathematics in Curriculum

Mathematics may be taken at all stages of the secondary school course, this being compulsory for the Intermediate Certificate course, that is, up to about 15 or 16 years of age. The age range of secondary school pupils is from 12 to 18 years.

About five hours a week is allocated to mathematics, or about three and a half hours in the case of roughly two thirds of the girls, who avail themselves of the possibility of taking elementary mathematics instead of the ordinary mathematics course during the first four years (i.e., up to the Intermediate Certificate Examination). Five hours represent from 17% to 20% of the total time-table of the schools, and three and a half hours proportionally less.

Mathematics is taught under the main headings of arithmetic, algebra, and geometry (including trigonometry). The subject is a required one in the first four years (i.e., up to the Intermediate Certificate Examination). It is optional, like all subjects except Irish, in the last two years (i.e., up to the Leaving Certificate Examination, pupils 16 to 18 years of age); during these two years, however, at least five subjects must be taken, and in the case of all boys and the great majority of girls mathematics is one of the five to nine subjects they choose to take.

A pass in arithmetic is essential in the secondary school entrance examination, and, for boys, a pass in mathematics in the Intermediate Certificate Examination. A pass in mathematics is not essential in the Leaving Certificate Examination; the National University of Ireland, however, requires such a pass (or a pass in mathematics in its matriculation examination), and in practice, as already noted, all boys and the great majority of girls take mathematics in the Leaving Certificate Examination.

Except for Irish, mathematics is the only subject in the Leaving Certificate Examination with a maximum of 600 marks (most other subjects have a maximum of 400 marks). This fact indicates the importance attached to mathematics.

Aims of Mathematics Teaching

The aims of mathematics teaching are not formally set down in any official instruction, for the reason that it would be extremely difficult to specify them. It is generally understood, however, that the practical aims are the cultivation of greater reasoning power and of greater accuracy in thought and expression, and that the cultural aim is the rounding off of the pupils' general education, as that education is described in the " Rules and Programme for Secondary Schools ".

Syllabuses

The ordinary mathematics syllabus of the Intermediate Certificate Examination covers the elementary mathematics syllabus of that examination, plus the following: arithmetic (compound interest, profit and loss, use of logarithms, elementary methods of approximate calculation, graphical treatment, algebraic methods); algebra (factors, identities, fractions, surds, remainder theorem, elementary theory of indices and logarithms, graphs of simple algebraic expressions); geometry (straight lines, triangles, rectangles, circles, roughly first four books of Euclid's Elements, algebraic proofs of theorems, elements of trigonometry).

For the Leaving Certificate Examination there is a pass course and an honours course in mathematics. The syllabus for the pass course covers fuller treatment of the Intermediate Certificate syllabus, plus the following: arithmetic (stocks and shares, mensuration of right prism, pyramid, cone and sphere); algebra (equations in one or two unknowns, progressions, graphic examination of simple algebraic functions); geometry (the substance of Book VI of Euclid's Elements, the theory and practical applications of trigonometrical properties, the use of tables).

For the Leaving Certificate Examination there is also a pass and an honours course in applied mathematics. The syllabus of the pass course covers velocity and acceleration, motion with constant acceleration, relative motion, parallel forces, motion on a smooth inclined plane; meaning of work, energy, power; equilibrium of a particle, friction, centre of gravity, liquid pressure, Archimedes principle, etc.

It is understood that an introduction to axioms and some elementary treatment of the history of mathematics should be part of the course, but this is not formally laid down.

Methods of Teaching Mathematics

There are no official instructions concerning methods of teaching mathematics in secondary schools, but it is the function of the inspector to ensure that what he deems effective methods are used. The most commonly employed methods are the blackboard, textbooks, oral questioning, and homework.

In regard to the question of the transfer stage from primary to secondary education, it is of interest to note that the inspectors and the Department frequently advise the assignment of the best teachers to the lower secondary classes, where it is felt that the solid foundations of the subject are laid. In respect of the correlation of mathematics with other subjects, it should be noted that one and the same inspector is responsible for both mathematics and science, and it is considered that this is one of the most effective ways of establishing such correlation; the schools follow the advice of the inspector in this connection.

Films, slides, etc., have not come into common use in the teaching of mathematics in secondary schools. The schools are at liberty to adopt such aids, but in practice many teachers prefer the traditional methods of instruction, and claim them to be more effective.

The mathematics syllabuses do not include applied mathematics, which is a separate subject and is taught only in the classes studying for the Leaving Certificate Examination.

The choice and use of particular textbooks are not stipulated by official regulation, but are matters for the individual school or teacher, subject to the advice of the Department's inspector during his visits.

Teachers

All secondary teachers, in order to be registered and recognised as such, and so to be entitled to State salary and emoluments, must possess a university degree and a post-graduate diploma in education. Most secondary teachers of mathematics have taken mathematics for at least one year of their university course, and many of them have taken their degree in mathematics.

There is some difficulty in recruiting fully qualified mathematics teachers. This is due to the fact that few students have a special bent for mathematics, and they tend to choose other subjects. Science graduates, moreover, whose university subjects usually include mathematics, seem to prefer to enter industry or vocational teaching.

Summer courses in the teaching of mathematics have been held from time to time by the Department of Education.

Miscellaneous

No major modifications will be introduced in the teaching of mathematics in secondary schools, until the Council of Education has completed the examination it is now making of the question, and has presented its report. There is no clear evidence that modern developments in the mathematical field have affected secondary education to any great extent. There is nevertheless a greater tendency than hitherto for university students to follow courses (e.g., engineering, physics, chemistry) in which mathematics is a basic subject, and in consequence somewhat more emphasis on mathematics and science subjects at the Leaving Certificate stage of secondary education. This increase of emphasis naturally tends to reduce the attention given to modern languages and arts subjects generally.

No formal experiments in the teaching of mathematics have been carried out by the Department of Education, but the Department's inspectors during their visits to the schools are expected to act as a pollenising influence in this regard.

ISRAEL

From the reply sent by the Ministry of Education and Culture

Place of Mathematics in Curriculum

The secondary course is a four-year one (grades 9-12 inclusive), the age range of the pupils being from 14 to 18 years. Mathematics is taught in all years and sections as a compulsory subject. The subject is designated algebra and geometry in the 1st and the 2nd year, trigonometry and solid geometry in the 3rd and the 4th year, and solid geometry, trigonometry, coordinate geometry, and calculus in the mathematics-physics section.

Four periods a week are allotted to mathematics, representing 11-12% of the total time-table.

Special importance is attached to mathematics in the mathematics-physics section of the 3rd and the 4th year. The subject is a compulsory one in the school leaving examinations.

Aims of Mathematics Teaching

The aims of teaching mathematics are to impart mathematic a techniques in solving mathematical and practical problems, and ability to reason logically and mathematically, and to prepare for scientific studies at higher level.

Syllabuses

The content of the secondary mathematics syllabuses is largely determined by the requirements of the secondary school leaving examinations. The main points in the mathematics syllabuses for the 1st and the 2nd year are as follows : algebra up to and including quadratic equations ; progressions, logarithms, compound interest, annuities ; plane or solid geometry ; trigonometry of the right-angled triangle, solution of triangles by the sine or cosine formulae. The main points in the mathematics syllabuses for the 3rd and the 4th year are as follows : algebra : series, mathematical induction, annuities ; trigonometry : sum and difference, formulas, identities, graphs of the functions, small angles, solution of triangles with the help of logarithmic tables ; coordinates, ellipse, hyperbola and parabola, areas and volumes of solids of revolution.

The secondary mathematics syllabuses do not as a rule include the history of mathematics. Whether this aspect of the subject is referred to or not depends on the teacher and the textbook used. In many schools two periods a week are devoted in the 3rd or the 4th year to the history of science, including mathematics. With regard to axioms, it is of interest

to note that an attempt was made to start teaching geometry with an introduction to them, but was not very successful. It is now proposed to give an introduction to axioms, as a review, after the completion of the geometry course. An experiment on the lines of this proposal, in a secondary school in Haifa, is referred to in a report on "The Teaching of Scientific Method in Secondary Schools", submitted to the Seventh International Congress on the History of Science (Jerusalem, August, 1953).

Methods of Teaching Mathematics

There is no uniformity in regard to the methods used for the teaching of mathematics at secondary level. The choice of method depends largely on the teacher's personality and training, and the intellectual level of the class. Very often model lessons are arranged, followed by general discussion among the participants. So far no final rules have been laid down with regard to methods.

The correlation of mathematics with other subjects in the secondary curriculum is best possible where there is one and the same teacher for the different subjects. In this connection, attempts are made to substitute the various riddles of the textbooks with practical problems of physics, chemistry, astronomy, etc.

The teaching materials used for mathematics lessons are generally limited to a modest set of models to visualize the various elements of solid geometry and thus to foster a concrete and real insight into space. In this respect there is still a great deal to be done, before full use has been made of all the technical means of the age. The notions of the elements of space are still taught abstractly and theoretically, as was the case many generations ago. The education authorities are convinced that the introduction of the film and the projector in the teaching of geometry could greatly affect, if not revolutionize, the results of such teaching, and they would welcome the appointment of a special committee of experts to devise an international set of models for such teaching.

The secondary mathematics syllabuses do not include applied mathematics, but mechanics is taught in the 3rd year as part of the physics syllabus.

Practical work goes hand-in-hand with the formal teaching, and includes the construction of geometrical models, the use of the sextant in measuring angles, distances, and heights, etc.

Textbooks have to be endorsed and recommended by the Ministry of Education and Culture, the requirements being clearness of exposition, and consistency with the syllabus and with scientific principles.

Teachers

Secondary teachers of mathematics are required to hold the Ph. D. or M. Sc. degree (in some cases holders of the B. A. or B. Sc. degree are allowed to teach the 1st and the 2nd year), and to have completed the course of a teacher training school or a one-year university course in psychology and pedagogy.

No difficulty is experienced in recruiting mathematics teachers. Serving teachers are encouraged to attend lectures and special courses, and the model lessons planned by the Ministry of Education and Culture and the association of mathematics teachers. A specialist journal is published for secondary teachers of mathematics.

Miscellaneous

It is proposed to teach mathematics in the non-mathematical sections during the first three years only.

Modern developments in the mathematical field have influenced the instruction given in the form of extra-curricular activities through special groups, but not class instruction.

ITALY

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

General secondary education [1] in Italy comprises a three-year lower cycle, the so-called *scuola media inferiore* (pupils from 10 to 13 years of age), and an upper cycle (pupils of 13 to 17 or 18 years of age) provided in the following three types of school: (a) classical secondary schools, with a five-year course; (b) science secondary schools, with a five-year course; (c) teacher-training secondary schools, with a four-year course.

Under the designation of *matematica*, mathematics is taught as a compulsory subject in all years of these various schools.

The weekly time allocation for mathematics is three hours (an average of 12.8% of the total time-table, practical activities not included) in the three years of the lower cycle, and as follows in the upper cycle:

Type of Sec. School	1st year		2nd year		3rd year		4th year		5th year	
	Hours	%	Hours	%	Hours	%	Hours	%	Hours	%
Classical	2	8	2	8	3	11.5	2	7.7	2	7.4
Science	5	21.7	4	16	3	11.5	3	11.1	3	10.7
Teacher-training .	4	14.2	3	10.3	2	7.7	2	8	—	—

It is recognized that mathematics is an important element in general education, and essential for the study of scientific and technical subjects but greater importance is not explicitly attached to it in any official publications, except in regard to science secondary schools.

Aims of Mathematics Teaching

At lower secondary level, the aim of teaching mathematics is to enable pupils to calculate rapidly, a skill which finds immediate application in the solution of practical problems. At upper secondary level, the subject becomes rational and systematic, and aims at imparting order, exactitude and soberness in speech, and a capacity for rigorous logical reasoning, analysis and synthesis, and the search for truth, and at helping to cultivate character and mental growth and discipline.

In the case of teachers-in-training, the revision in their final year of the principal points of arithmetic and geometry is undertaken mainly with a view to studying the method of teaching mathematics.

[1] The present inquiry does not cover the vocational and other specialized secondary schools.

Syllabuses

Lower secondary cycle. Practical arithmetic and plane geometry ; in the 3rd year, elementary algebra and first-degree equations, and spatial geometry.

Classical secondary schools. 1st year. Algebraical calculation, plane geometry. — *2nd year.* Algebraical calculation, as far as the solution of first-degree equations ; plane geometry, as far as the study of congruence. — *3rd year.* Algebra : systems of equations of the second and higher degrees. Geometry : proportional sizes, similar figures. — *4th year.* Algebra : progressions, exponential equations, logarithms. Geometry : the circle, spatial geometry. — *5th year.* Trigonometry, spatial geometry.

Science secondary schools. 1st year. Algebra : algebraical calculation, first-degree equations with one unknown. Geometry : geometrical elements, theory of congruence of plane figures. — *2nd year.* Algebra : real numbers and powers, systems of second-degree equations, progressions ; graphical representation of functions. — *3rd year.* Algebra : exponential functions, logarithms, use of slide rule, spatial geometry. — *4th year.* Plane trigonometry, derivatives ; equivalences, calculation of areas and volumes. — *5th year.* Integral calculus, combination.

Methods of Teaching Mathematics

Teaching at the lower secondary level is of an intuitive and experimental kind. It continues to make use of intuition at upper secondary level, but at the same time becomes rational and systematic.

JAPAN

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Secondary education in Japan is provided in (a) lower secondary schools, with a three-year course which is compulsory for all pupils, boys and girls alike, on completion of the six years of compulsory primary schooling, and (b) upper secondary schools, with a three-year full-time course and a four-year part-time course ; neither of these courses is compulsory. Mathematics is a required subject in lower secondary schools, and is offered in both the full-time and the part-time course in upper secondary schools.

The pupils are 12, 13 and 14 years old in the 1st, the 2nd, and the 3rd year respectively of lower secondary schools, and 15, 16, 17 and 18 in the 1st, the 2nd, the 3rd, and the 4th year respectively of upper secondary schools. In lower secondary schools, the time allocation for mathematics is four or five hours a week (i.e., 12-17% of the total time-table of thirty to thirty-four hours a week) in the 1st year, and three to five hours (9-17%) in the 2nd and the 3rd year.

In upper secondary schools the credit system is in operation. The various " subject-areas " are mostly divided into a number of subjects, and each of these is allotted a certain number of credits (a credit of " 1 " involves instruction for one hour a week throughout the school year of thirty-five weeks). For graduation, each pupil is required to take at least 85 credits ; pupils ordinarily take 90-96 credits.

The mathematics subject-area is divided into Maths. I (6 or 9 credits), Maths. II (3 credits), Maths. III (3 or 5 credits), and Applied Maths (3 or 5 credits) ; the content of these four subjects is given below in the " Syllabuses " section.

Maths. I is assigned for all pupils in the 1st and the 2nd year of upper secondary schools ; the six or nine hours' instruction a week for one school year, corresponding to its allotted 6 or 9 credits, is distributed in various ways. Maths. II is assigned for pupils who need it, after they have taken at least 6 credits for Maths. I, i.e., in the 2nd year or later. Maths. III is assigned for pupils who need it, after they have completed Maths. II, i.e., in the 3rd year. Applied Maths. is assigned for pupils who need it, after having completed Maths. I.

Each upper secondary school offers one or both of the following two courses : (a) Maths. I, Maths. II, and Maths. III ; (b) Maths. I, Maths. II (if necessary), and Applied Maths. Course (b) is deemed especially appropriate for vocational upper secondary schools. In practice, the choice of offerings depends on the individual needs of the pupils, and the type of school.

In vocational upper secondary schools, Maths. II and III are optional, and Applied Maths. is required, for pupils preparing for technical vocations, while all three are optional for pupils preparing for non-technical vocations.

No special importance is attached to mathematics, as compared with other subjects.

Aims of Mathematics Teaching

The general objective assigned to mathematics teaching is appreciation of the usefulness and beauty of mathematics, and of the part it plays in cultivating awareness of truth, righteousness, and justice, in saving labour and time, and in the growth of independent thought. Mathematics teaching, moreover, should aim at cultivating understanding of basic mathematical concepts and principles, accuracy, efficiency in the use of quantitative procedures, and appreciation of the social and civic value of mathematical abilities and attitudes.

The objectives assigned to mathematics teaching in lower secondary schools are ability to generalize, use formulas, make approximations and estimate the degree of error, and use number as a practical basis of prediction, and appreciation of the significance of mathematics both to the individual and to society, and in any situation (scientific and technical research, social cooperation, labour-saving) calling for informed reasoning and valid conclusions.

In upper secondary schools the aims of mathematics teaching are comprehension of the basic concepts, laws, and principles of mathematics, and capacity to use and formulate data accurately, to reason, to reach conclusions, and to grasp the nature of proof and the necessity for disciplined logic.

Syllabuses

Lower secondary schools. 1st year. Fractions, interest, weights and measures, areas and volumes, elements of geometry. — *2nd year.* Revision of previous year's work ; algebra (elements, equations in one unknown) ; geometry (formulas for measuring area and volume of various figures) ; use of instruments (graphs, drawing figures). — *3rd year.* Revision of previous year's work ; proportions, equations in two unknowns, graphical interpretation of linear relations, theorem of Pythagoras and applications, square root, elements of trigonometry ; appropriate practical work (drawings, compound interest, etc.).

Upper secondary schools. Maths. I. Algebra : functions, the four operations, factors, equations, logarithms, statistics. Geometry : properties of linear figures, the circle, loci, solid figures. Trigonometry : sine, cosine, etc. The central ideas to be emphasised are mathematical symbolism and formalism, the nature of proof, interdependence, invariant relations, and the relations between geometry and algebra. — *Maths. II.* Fractional and irrational equations, theorems ; functions (functions of the third degree, logarithmic and exponential functions), trigonometrical functions (periodicity, theorems), geometrical figures and their equations (straight lines, the circle, conic sections), etc. — *Maths. III.* Sequences and series, differential and integral calculus, permutations and combinations, probability and statistics. — *Applied Maths.* Statistics, sequences and series, complex numbers and their geometrical representation, trigonometrical functions, trigonometry applied to surveying, differentiation, integration, calculations applied to industry, geometrical figures and their equations.

The mathematics syllabuses do not include the history of mathematics, but most of the textbooks in use do so. An introduction to axioms is included in the syllabus of Maths. I.

Methods of Teaching Mathematics

Official suggestions concerning methods of teaching mathematics at secondary level have been issued. They put forward the following principles: the development of pupil understanding by proceeding from the concrete to the abstract; the development of pupil appreciation through significant experiences involving the use of mathematics for achieving exactness, accuracy, and efficiency; the organization of learning experiences by the unit method. The last of these three principles is now much criticized, and is not much practised.

Two paragraphs appear in the same official source, suggesting a careful and gradual transition to more logical treatment of contents during the transfer stage from primary to secondary education. So far as the correlation of mathematics with other subjects is concerned, this is a matter which is left to the initiative of the teachers; the mathematics syllabuses give emphasis to applications of mathematics to, and use material from, other subjects, but there is no formal correlation in this respect.

The teaching materials and auxiliary aids used for mathematics lessons include models of geometrical figures, and measuring instruments, and some films and slides.

As already indicated, applied mathematics forms a separate subject, dealing with the mathematical aspects of various technical subjects.

Teachers must use textbooks authorized by the Ministry of Education. Such authorization is based on official requirements in respect of up-to-dateness, organization, style, and adaptation to the level of the pupils.

Teachers

The qualifications secondary teachers of mathematics are required to possess, are laid down in an educational personnel certification law. Such teachers must hold a least a first degree, and must have taken a number of credits varying according to their teacher-category (first class, second class) and whether they teach at lower or upper secondary level.

Great difficulty is experienced in recruiting mathematics teachers; about 9% of such teachers in upper secondary schools are without the required qualifications, and about 25% in lower secondary schools. The higher percentage in the case of lower secondary schools is due to the great increase of enrolments following the school reform begun in 1947. Credit-giving lectures and correspondence courses are being organized to overcome the difficulty.

Secondary teachers of mathematics have various opportunities of obtaining further training. There are facilities for in-service training at university level; workshops and conferences are conducted by the Ministry of Education, the local and regional education authorities, and the associations of mathematics teachers; a number of educational journals are available, including the monthly "Journal of Japan Society of Mathematical Education".

Miscellaneous

In the period 1948-1954 the upper secondary schools adopted an elective system whereby the mathematics subject-area was organized into four subjects : general mathematics, algebra I, algebra II, and geometry. The arrangement was criticized, however, as being unlikely to give pupils a systematic understanding of mathematics as a whole, and the system as detailed above was adopted in 1955, for application in 1956. So far as the lower secondary schools are concerned, the grade placement of the content of the mathematics syllabuses has been and still is the object of widespread criticism, on the grounds that the requirements of the compulsory aspects of the course are too low, and on other grounds.

Modern developments in the mathematical field have influenced secondary mathematics teaching in a number of ways. During the past thirty years, increasing attention has been given to the concept of function ; graphic representations of data of various kinds are emphasized ; the use of the slide rule is now taken in lower secondary schools ; an understanding of the principles of axioms is now recognized as one of the essential qualities of mathematics teachers at upper secondary level ; elementary calculus and the concept of probability and statistics have been introduced into the syllabuses of the upper secondary schools.

Experimental studies concerning the teaching of mathematics are being carried out. They mainly concern adaptation of that teaching to differences in pupils' aptitudes, the major causes of pupils' errors and ways of preventing them, and assisting pupils to solve problems by means of graphical representation.

LAOS

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

The secondary course lasts seven years, the age range of the pupils being from about 14 to 20 years, and comprises a four-year lower and a three-year upper cycle. Mathematics is taught, under that name, as a compulsory subject throughout the course ; the number of hours allotted to it increases gradually from two hours a week in the 1st year to nine hours in the mathematics class in the 7th year. In addition, pupils in the classical section of the upper cycle are offered optional, supplementary teaching in mathematics.

Mathematics is regarded as one of the most important subjects in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The main aims of mathematics teaching at secondary level are to inculcate the scientific attitude, the basis of education, and to serve as an introduction to abstract reasoning and as preparation for scientific study at higher level.

Syllabuses

1st year. Arithmetic. — *2nd year*. Arithmetic. Geometry : study of the triangle. — *3rd year*. Arithmetic. Geometry : triangles, parallel straight lines, the circle. Algebraical calculations. — *4th year*. Arithmetic. Algebra : polynomials, linear functions, numerical equations of the first degree with one unknown. Geometry : loci, similar triangles, orthogonal projections, regular polygons, areas. — *5th year*. Algebra : algebraical numbers, vectors, functions, problems of the first degree, second-degree equations. Geometry : plane figures, triangles, the circle, ratios and loci, elements of trigonometry. — *6th year*. Algebra : trinomials of the second degree, derivatives. Geometry : solid geometry, orthogonal projections, lateral areas and volumes, elements of descriptive and of projective geometry. Trigonometry : circular functions, equations, problems. — *Mathematics class*. Theoretical arithmetic. Algebra : functions, equations, derivatives, logarithms. Trigonometry : equations, functions, solution of triangles. Kinematics : vector speed, vector acceleration, study of movements. Statics : notion of mass, equilibrium. Geometry : revision and completion of work done in previous years, descriptive and projective geometry, cosmography.

The mathematics syllabuses also include some reference to the history of mathematics.

Methods of Teaching Mathematics

Suggestions as to suitable methods of teaching mathematics have been officially issued, recommending teachers to make every endeavour to understand the difficulties encountered by their pupils, in order to be in a position to facilitate the work and lay a sure foundation for future studies.

Teachers do all in their power to smooth the transition from primary to secondary education, during their work in the 1st year and in some cases also in the 2nd year. They revise most of the mathematics syllabus for the final primary class, and, while continuing to make their teaching concrete, gradually lead their pupils to think for themselves, and to proceed logically instead of merely automatically and mechanically.

In regard to the coordination of mathematics with other subjects in the curriculum, it should be noted that the mathematics and the physics teachers keep in close touch with each other, and decide in what order given notions shall be introduced.

The materials used for teaching mathematics include the ordinary materials used for geometrical drawing and constructions, in all years, but not films or filmstrips.

The practical applications of mathematics consist of geometrical and industrial drawing, in the intermediate classes, and the making of drawings of descriptive geometry, in the mathematics class in the 7th year.

The textbooks used for mathematics are those used in the secondary schools in France.

Teachers

Secondary teachers of mathematics are required to have taken the *agrégation* or a first degree in a university in France.

Some difficulty is experienced in recruiting mathematics teachers, but less than in the case of physics teachers, who are drawn into industrial undertakings.

Mathematics teachers are able to improve their knowledge by studying the specialist journals, in particular two educational journals published in France, *Journal de mathématiques élémentaires*, and *L'éducation mathématique*.

Miscellaneous

No changes are at present envisaged in the secondary mathematics syllabuses. The syllabus for the final year has been influenced by modern developments in the mathematical field, and lays considerable stress on vectorial methods.

LIBERIA

From the reply sent by the Department of Public Instruction

Place of Mathematics in Curriculum

The secondary course is a four-year one, following an eight-year primary course. The age range of the pupils is from 14 to 22 years. Mathematics is taught as a compulsory subject throughout the secondary course. Three and three quarter hours a week are allocated to the subject. Mathematics is designated " general mathematics and algebra " in the 1st year, and " algebra and geometry " in the following years.

The determining credits for graduation from a secondary school are obtained from the accumulated results of monthly tests and semester examinations during the four years of secondary education. Mathematics ranks as a major subject. Leaving examinations are not held.

Aims of Mathematics Teaching

Mathematics is taught with a view to establishing a good foundation and to developing speed and accuracy, in the 1st year, and to developing habits, neatness, order, diligence, and honesty, in the remaining years.

Syllabuses

1st year. General mathematics (first semester) : accuracy and skill in handling the fundamental operations of arithmetic ; applications to such topics as the family budget, bank accounts, cash account, personal accounts, percentage, interest. Algebra (second semester) : the fundamental operations, simple equations. — *2nd year.* Algebra : the use of equations in solving problems, brackets, fractions, multiplication and division of monomials, simultaneous linear equations. Geometry : elementary notions (learning by doing), geometric constructions, formulas of areas, square roots. — *3rd year.* Algebra : literal numbers, positive and negative numbers, equations containing parentheses and fractions, graphs, systems of linear equations, special products and factoring, fractions, powers, roots, radicals, quadratic equations. Geometry : parallels, angles, triangles, trigonometry of the circle. Numerical trigonometry : angles, triangles, sine, cosine, tangent, interpolation. — *4th year.* Algebra : systems of equations with problems, indices and logarithms, arithmetical and geometrical progression. Geometry : the congruency of triangles, parallelograms, mid-point theorem, areas of quadrilaterals and triangles, extension of the theorem of Pythagoras, the theorem of Appolonius ; theorems on circles, arcs, and chords ; constructions on circles, tangents, and polygons. Numerical trigonometry : tangent, cotangent, secant, cosecant, sine and cosine rules, solution of triangles, identities.

The syllabuses also include the history of mathematics and an introduction to axioms.

Methods of Teaching Mathematics

There are no oficial instructions regarding methods of teaching mathematics. At the transfer stage from primary to secondary education, stress is laid on developing the pupils to the point where they begin to develop the principles of reasoning from the concrete to the abstract, from the known to the unknown, on accuracy, which shows a respect for logical reasoning, and on speed, orderly arrangement, etc.

The science subjects, as for example physics, chemistry, economics and biology, are subjects which necessarily involve mathematical calculations, and are therefore closely correlated with the teaching of mathematics.

The materials used for mathematics lessons include set-squares, compasses, rulers (marked in inches and centimetres), protractors, globes, chronometers, microscopes, telescopes, metronomes, gramophones, radios, etc.

Formal teaching is introduced by practical work of various kinds, and the knowledge acquired is applied practically.

Official instructions have been issued concerning the choice and use of mathematics textbooks. Such textbooks are required to give a progressive and comprehensive treatment of the subject, and to be lucid, up-to-date, interesting and thought-challenging, and within the pupils' powers of appreciation.

Teachers

The Department of Public Instruction requires secondary teachers to hold the B. A. or B. Sc. degree in education awarded by the teacher training school of the University of Liberia.

There is some difficulty in recruiting secondary teachers of mathematics. This is mainly due to the fact that in the past there were few scholarships for higher training abroad, owing to lack of financial means, and mathematics teaching did not offer sufficient inducements. At the present time, however, the financial position is such that scholarships are available for higher studies in mathematics, and sufficient inducement can be offered to young mathematics graduates to take up teaching.

To keep serving teachers of mathematics up-to-date in their subject, the government employs the system of workshop instruction during the vacation period.

Miscellaneous

The Department of Public Instruction is contemplating certain changes in the teaching of mathematics. The present syllabuses lay stress on preparation for study at higher level, and to some extent neglect certain basic ideas forming the groundwork for many kinds of occupations. The envisaged changes will make added provision for the basic skills and knowledge enabling secondary pupils to take up such occupations.

LUXEMBURG

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

Mathematics is taught in all years of secondary education, except the final year of the commercial section of the modern course.

The weekly time allocation, and the percentage this allocation represents of the total time-table, are given in the following table :

A. *Boys' Secondary Schools.* (a) Classical course lasting seven years (classes VII to I, pupils from 12 to 18 years of age) :—

	Hours	%
1st, 2nd, 3rd & 4th years	3	10.7
5th year :		
Greek-Latin section	3	11.1
Latin sub-section A [1]	3	10.7
Latin sub-section B	5	18.5
Latin sub-section C	3	10.7
6th year :		
Greek-Latin section	3	11.1
Latin sub-section A	3	10.0
Latin sub-section B	5	18.5
Latin sub-section C	4	10.4
7th year :		
Greek-Latin section	3	11.1
Latin sub-section A	3	10.7
Latin sub-section B	6	22.2
Latin sub-section C	4	10.5

(b) Modern course lasting six years (classes VI to I, pupils from 12 to 17 years of age) : 1st year, 3 hours (10.7%) ; 2nd year, 4 hours (14.8%) ; 3rd year, 5 hours (19.6%) ; 4th year : industrial section [2], 6 hours (24.0%), commercial section, 3 hours (11%) ; 5th year : industrial section, 7 hours (28%), commercial section [3], 2 hours (7%) ; 6th year : industrial section, 8 hours (33.3%).

[1] The Latin sub-section A prepares pupils for philological studies, the Latin sub-section B for scientific careers, and the Latin sub-section C for careers in natural science, medicine and pharmacy.

[2] The industrial section prepares pupils for intermediate technical studies and gives access to the higher technical schools.

[3] The commercial section prepares pupils for commercial careers and for the higher schools of commerce.

B. *Girls' Secondary Schools.* The course lasts seven years (classes VII to I, pupils of from 12 to 18 years of age) :—

	Hours	%
1st, 2nd & 3rd years	3	10.1
4th year	3	10.0
5th year	3	10.7
6th year	3	10.0
7th year	3	10.7

Mathematics is designated arithmetic in the 1st and the 2nd year, and mathematics in the remaining years.

Mathematics is a compulsory subject for all pupils, and ranks as a main subject on an equal footing with languages. In the classical course the mark given for mathematics is the average of the marks obtained for algebra, geometry, and trigonometry ; all these aspects of the subject are taught by the same teacher. At the upper stage of the modern course, algebra, analytical geometry, and descriptive geometry are considered to be separate subjects, and are taught by different teachers. A pupil obtaining insufficient marks in two main subjects may not be promoted.

Aims of Mathematics Teaching

According to the official instructions on methods " the teaching of mathematics aims at contributing to the pupils' education through the development of their sense of the logical, their intuition, and their aptitude for research. It should in addition give them the sound mathematical knowledge called for by modern civilization."

Syllabuses

A. *Boys' Secondary Schools.* I. *Classical course. Lower stage* (1st and 2nd year). Arithmetic : decimal numeration ; the four fundamental operations with whole numbers and decimal numbers ; highest common factor and lowest common multiple ; fractions ; the metric system ; simple interest, discount ; stocks and shares, mixtures and alloys. — *Intermediate stage* (3rd and 4th years). Algebra : algebraic symbols ; exercises on algebraical calculation ; first-degree equations in one and more unknowns ; resolution into factors ; fractions ; square roots ; second-degree equations with numerical coefficients. Geometry : theory of parallels ; congruence of triangles ; properties of the parallelogram ; the circle ; inscribed and circumscribed quadrilaterals ; problems. — *Upper stage* (5th, 6th and 7th years). (a) Greek-Latin section and sub-sections A and C. Algebra : properties of the roots of second-degree equations ; indices ; negative and fractional exponents ; radicals ; logarithms ; compound interest ; annuities ; the concept of function ; graphical representation of straight lines, circles, and conic sections given by a reduced equation ; algebraical and graphical study of second-degree simultaneous equations in two unknowns ; the limit and continuity of a function ; derivatives ; problems

involving maxima and minima ; the concept of the primitive, infinite integral and finite integral. Geometry : similar triangles ; relationships in the dimensions of the elements of the triangle and the circle ; solid geometry (straight lines, planes, polyhedral angles, the prism, the pyramid, the cylinder, the cone, the sphere, calculation by integration of simple plane areas and of the volume of common solids of revolution). Plane trigonometry : definition of trigonometrical functions ; operations with arcs ; calculations with logarithms ; solution of trigonometrical equations in one unknown ; solution of triangles (principal cases). (b) Latin sub-section B. Same syllabus as for the other sections, plus the following. Algebra : discussion of linear systems with several variables ; determinants ; complex numbers ; analysis ; Newton's binomial theorem ; series ; rules of differentiation ; methods of integration. Analytical geometry : straight lines ; the circle ; conic sections ; transformation of axes ; tangents and normals ; centre, diameter, axes and asymptote of conical sections ; introduction to polar coordinates. Descriptive geometry (Monge method) : representation of points, lines, and planes ; intersection of two planes ; point of interception of a line and a plane ; relative position of lines and planes ; projections. Geometry : elements of vectorial geometry ; plane transformations, homologous transformations, inversion ; polarities ; radical axes ; pencil of circles. — II. *Modern course. Lower stage.* 1st and 2nd years. Arithmetic : practically the same syllabus as for the 1st and the 2nd year of the classical course. 3rd year. Algebra : algebraical calculation ; resolution into factors ; fractions ; first-degree equations in one and more unknowns ; proportional parts, interest, etc. Geometry : fundamental concepts ; congruence of triangles ; theory of parallels ; properties of parallelograms ; the circle, arcs, angles ; principal points and lines of a triangle ; constructions and problems. — *Upper stage* (4th, 5th and 6th years). (a) *Industrial section.* The syllabus is practically the same as that of the Latin sub-section B of the classical course, except that is includes in addition spherical trigonometry and the geometrical study of conic sections in the 5th year, and takes the work on descriptive geometry as far as the intersection of two surfaces in the 6th year. (b) *Commercial section.* Algebra : first-degree equations and inequalities ; determinants ; solution and discussion of second-degree equations ; complex numbers ; graphical study of linear functions and second-degree trinomials ; systems of simple second-degree equations ; indices of exponents of all categories ; simple exponential equations ; logarithms ; progressions ; Newton's binomial theorem and its application. Geometry and trigonometry : proportional segments ; similar triangles ; relations in the dimensions of the triangle and the circle ; definition of trigonometrical functions ; solution of triangles (principal cases) ; use of tables of natural values of trigonometrical functions.

B. *Girls' Secondary Schools.* The syllabuses are the same as those of the Latin sub-section A of the classical course in boys' secondary schools. In addition, the mathematics syllabus of the special optional course for pupils intending to study natural science or medicine covers the following : introduction to vectors and analytical geometry ; derivatives and simple applications of them ; primitives and simple applications taken from the calculation of areas and from physics.

The secondary mathematics syllabuses do not refer to the history of mathematics, but the official instructions on methods recommend teachers to give historical details and to speak of the importance of certain questions in the history of mathematics.

No provision is made for any systematic introduction to the principles of axioms.

Methods of Teaching Mathematics

The official instructions on methods state as follows in regard to the method to be used in teaching mathematics: " In this subject, more than in any other, learning means understanding. Teachers should therefore ensure that all abstract ideas have been properly understood. Such ideas must for this reason be introduced by means of concrete and easily grasped examples. The manner of presentation will in general be the heuristic one, which makes teaching living and stimulating. The language used, in conformity with the exactitude of mathematics, must be simple and precise."

During the transition from primary to secondary education, teachers are expected to take due account of the difficulties arising through the change-over from German to French as medium of instruction. They must base their work on the knowledge that the pupils have already acquired, and, while taking the primary syllabus a stage further, must gradually introduce abstract and rational elements into their lessons, alongside the concrete and intuitive elements which must predominate at this stage.

Coordination between mathematics and other science subjects is ensured by the syllabuses and textbooks. Very often the same teacher takes physics and mathematics; when this is not the case, the teachers concerned ensure that their work is coordinated by arranging discussions, lectures, etc., often under the direction of the headmaster of the school.

Moveable figures and models that can be taken to pieces are often used for the teaching of arithmetic, geometry, descriptive geometry, and trigonometry. Every school is equipped with projectors for films and slides, but such aids are not used to any considerable extent for mathematics, and a film concerning the subject is shown only when it may serve to synthesize and draw together ideas and properties already taken in class.

Mathematics being regarded as a subject whose main purpose is to contribute to the pupils' general education, exercises in surveying and handicraft work are not given. This does not imply, however, that modern methods are not used in teaching the subject. A mathematics class is run on activity lines; the pupils take part in the proof of theorems, the solution of problems, and the construction of figures. In choosing problems, teachers give preference to concrete texts which are related to the pupils' experience and to practical applications. The solution of problems nevertheless takes place wholely within the classroom, and never gives rise to practical work in field or workshop.

Regulations concerning the use of textbooks are contained in the official time-tables and syllabuses issued by the education authorities at at the beginning of each school year. Such use is compulsory, and as a result the lessons given in one school are in complete conformity with

those given in any other school. The arrangement is necessitated by the fact that the organization of promotion and school leaving examinations is such that all the pupils of a given type of school must have the same curricula and syllabuses.

Teachers

Secondary teachers of mathematics are required to have completed (a) eight semesters of university study, of which two may be accomplished in Luxemburg (higher courses, the physics and mathematics section), and (b) two years of practice teaching. That part of the university study which is done abroad, need not necessarily conclude with an examination ; candidates are in any case required to appear before a Luxemburg degree examination board. Teachers-in-training are awarded the title of *candidat* after six semesters of university study, and the doctorate in physics and mathematics after eight semesters. After qualifying, each candidate must teach for two years in a secondary school selected by the school authorities.

No special difficulties are met with in regard to the recruitment of secondary teachers of mathematics. For some years the number of applicants for posts has exceeded the demand. In view, however, of the increase in enrolments, this position is not to be deplored.

The teachers follow with care and interest the development of the teaching of mathematics in neighbouring countries. No textbooks are published in Luxemburg, and use has therefore to be made of foreign ones. The teachers familiarize themselves with modern trends through reading journals and books published abroad, which they obtain for themselves, or secure from the school library or the national library in the capital.

Further training for mathematics teachers is not provided by their professional association. The government gives them every encouragement to undertake study abroad, wherever justified by their work and the needs of their school.

Miscellaneous

As a result of a comparatively recent reorganization of the upper stage of the classical course, more emphasis was given to mathematics, physics and chemistry in the Latin sub-section B. Elements of differential and integral calculus were added to the mathematics syllabus of the final year of the Lation sub-section A.

The revision of the descriptive geometry course for the industrial section is now under being considered. Progression revision is taking place of the mathematics syllabuses for girls' secondary schools, in order to shorten the arithmetic course from three to two years, to make room for a more advanced study of the concept of function and graphical representation, and to introduce elements of differential and integral calculus in the final year.

MONACO

From the reply sent by the Ministry of State

Place of Mathematics in Curriculum

Under the designation of *mathématiques*, mathematics is taught in all years and sections of the general secondary school, the course of which is a seven-year one concluding with the baccalaureate examination. The age range of the pupils is from 11 to 18 years.

The hours per week allotted to the subject are as follows: 1st year, two hours (classical and modern sections); 2nd and 3rd year; two hours and a half (classical and modern sections) and in the 3rd year, three hours (short modern section); 4th year, two hours and a half (classical and modern sections) and three hours (short modern section); 5th year, one hour and a half (classical sections A and B) and four hours (classical section C and modern section); 6th year, one hour and a half (classical sections A and B) and four hours (classical section C and modern section); 7th year, one hour and a half (philosophy class), four hours (experimental science class), and nine hours (mathematics class).

All the hours indicated above are compulsory. In addition, one and a half optional hours are allotted to mathematics in classical section B in the 5th and the 6th year.

The importance attached to mathematics, of course, in comparison with other subjects, is greater in the Latin-science and modern sections than in the classical section.

Aims of Mathematics Teaching

The two main aims of mathematics teaching at secondary level are intellectual training, and preparation for specialist studies at higher level.

Syllabuses

1st year. Calculations with whole numbers and decimals, the decimal system of measurements, the rule of three, the measurement of time, percentages, simple interest. Geometry: the measurement of angles (use of protractor), area of polygons and circles, volumes. — *2nd year.* The fundamental operations, fractions, introduction to algebra, concrete problems leading up to equations of the first degree with one unknown, equation of problems. Geometry: elements of geometry, straight lines, angles, plane figures, congruency of triangles, graphical constructions. — *3rd year.* Prime factors, the highest common factor and the lowest common multiple; algebraical numbers and operations, algebraical expressions, equations of the first degree with one unknown. Geometry: study of

triangles (congruency and inequality), properties of parallel straight lines and perpendiculars, properties of parallelograms, dimensional relations of the circle, constructions. — *4th year.* Ratios and proportions, proportional parts, square root. Polynomials, principal identities, notions of variables and functions ; study of the linear function and graphical representation ; equations and inequalities of the first degree with one unknown, impossibility and indeterminateness. Geometry : notions of reciprocal theorems, loci, problems on construction ; ratios of segments, the theorem of Thales, similarity of triangles, dimensional relations of the triangle, orthogonal projections, sine, cosine, tangent, trigonometrical relationships ; inscribed and circumscribed circles. — *5th year (sections A and B).* Algebra : revision of the syllabus for the previous year ; the function of a variable, graphical representation ; systems of first-degree equations, equations of the second degree with one unknown, applications. Geometry : similar triangles, dimensional and trigonometrical relations in the right-angled triangle, revision of the syllabus of the previous year. — *5th year (section C and modern section).* Algebra : As for the 5th year, sections A and B, and, additionally, powers, ratios, and operations on polynomials and rational fractions. Geometry : straight lines and demi-planes, angles of a polygon, parallelograms, translation ; the circle ; similarity, homology ; harmonic division ; dimensional and trigonometrical relations in the right-angled triangle ; circular functions (tables) ; regular polygons. — *6th year (sections A and B).* Algebra : general equations of the second degree with one unknown, roots ; definition of the derivative. Geometry : planes and straight lines, dihedral angles, polyhedrons ; volumes, areas, prismatic sections, pyramids, cylinders, cones and spheres. — *6th year (section C and modern section).* Algebra : as for the 6th year, sections A and B, and, additionally, inequalities of the second degree, rectilinear movements. Trigonometry : arcs and angles, circular functions. Geometry : as for the 6th year, sections A and B, and, additionally, equipollent vectors, translation, projection and revolution of the plane of projection. — *Philosophy class.* Trigonometry and algebra : arcs and angles, circular functions, derivatives, prime functions. Cosmography : the heavens and the constellations, the earth, the solar system, the moon, the stars, the Milky Way. — *Experimental science class.* Arithmetic : resolution into prime factors, fractions, combinations, simple probabilities. Algebra and trigonometry : derivatives, logarithms, arithmetical and geometrical progressions, vectors, arcs and angles, solution of some trigonometrical equations, the function of a variable, definitions and graphical interpretation of derivates and differentials. Mechanics : the kinematics syllabus of the mathematics class. Cosmography : as for the philosophy class. — *Mathematics class.* Arithmetic : as for the experimental science class, and, additionally, squares and square roots. Algebra : algebraical calculation, increasing and decreasing functions, rectangular coordinates, equations and inequalities of the first and the second degree, graphical representations, notion of the prime function (calculation of area), progressions, logarithms, compound interest. Trigonometry : revision, the sum and difference of 2 sine and of 2 cosine, solution of triangles. Kinematics : the relative motion of a point, uniform rectilinear and circular motions, speed and acceleration, angular speed, translation and rotation of a solid. Statics : mass, the material point, vector-force, composition of forces, equilibrium of a material

point, reaction. Geometry : revision and exercises, projections, systems of axes of coordinates, plane and solid geometry (congruence, symmetry, rotation and translation, areas and volumes, similarity, homology, harmonic division, the circle, the sphere, cones, plane sections of the cone and the cylinder, various loci). Descriptive and projective geometry : the point, the straight line, the plane, intersection, orthogonality, change of plane, axial rotation, revolution of the plane of projection, angles, prisms, pyramids, the projection of a circle. Cosmography : as for the philosophy class, but to a more advanced level.

The official syllabuses do not cover the history of mathematics or an introduction to the principles of axioms.

Methods of Teaching Mathematics

Suggestions as to suitable methods of teaching mathematics have been issued. The main recommendation they make is that teachers should proceed from the concrete to the abstract. No special steps have been taken to smooth the transition from primary to secondary education, but in the 1st year about half the hours allotted to mathematics are devoted to a revision of the mathematics syllabus of the final primary class.

The teaching of mathematics is coordinated with that of other subjects. In each year the physics syllabus, for example, includes practical applications of the work done in mathematics.

The teaching materials used for mathematics include films, filmstrips, and various kinds of apparatus, including models of the planetary system.

The textbooks used are chosen by the teachers.

Teachers of Mathematics

Secondary teachers of mathematics are required to have passed the *agrégation* examination and to hold the secondary teacher's certificate *(C.A.P.E.S.)*. No difficulty is experienced in recruiting them. They are able to keep abreast of progress made in the field of mathematics, and to raise the standard of their teaching, through the study of specialist journals.

NEPAL

From the reply sent by the Ministry of Foreign Affairs

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the six years of secondary education. The age range of the pupils is from 11 to 16 years. Four and a half hours a week is allocated to the subject (13-14% of the total time-table).

There are optional classes in mathematics in the last two years of the secondary course.

Special importance is attached to mathematics in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The aims of teaching mathematics at secondary level are to impart fundamental ideas of mathematics and to prepare the pupils for higher studies in various technical subjects.

Syllabuses

The mathematics syllabuses now in force are designed to prepare pupils for the secondary school leaving certificate examination, which gives access to higher education.

1st and 2nd years (classes V and VI). Arithmetic : the four fundamental operations (simple and complex number), prime factors, fractions. Algebra : the four operations. Geometry : the drawing and measurement of straight lines, axioms. — *3rd year* (class VII). Arithmetic : vulgar and decimal fractions, square root, exercises, unitary method. Algebra : substitutions, simple formulas and their applications. Geometry : elementary theorems on angles and triangles. — *4th year* (class VIII). Arithmetic : revision of the work done in the previous year ; interest. Algebra : revision of the work done in the previous year ; simple equations, fractions. Geometry : theorems on angles, triangles, and parallels ; drawing and measurement of angles and triangles. — *5th year* (class IX). Arithmetic : revision of the work done in the previous year ; time and work ; pipes and cisterns ; square root. Geometry : theorems on angles, triangles, parallels, parallelograms, areas, chords, angles in a circle ; the construction of triangles and quadrilaterals ; loci. — *6th year* (class X). Arithmetic : operations on fractions, the metric system, ratio and proportion, simple interest, square measure and cubic measure. Algebra : ratios, equations, simple graphs. Geometry : revision of the work done

in the previous year ; tangents, geometrical equivalents of some algebraical formulas, experimental demonstrations of and deductions from the theorems studied, contructions.

The secondary mathematics syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

The education authorities have not issued any special instructions concerning the methods to be used for the teaching of mathematics. Teachers generally make use of both theoretical and practical procedures.

Teachers endeavour to correlate the teaching of mathematics with subjects such as geography and science. No provision is at present made for applied mathematics.

Modern materials for the teaching of mathematics are not available. No special instructions have been issued concerning mathematics textbooks ; the best available textbooks are selected.

Teachers

Mathematics is taught at secondary level by persons holding a degree or an intermediate certificate from an Indian university. No special training is required. No difficulty is experienced in recruiting mathematics teachers.

Miscellaneous

The education authorities intend to introduce the direct method of teaching mathematics, with modern equipment, and to increase the provision of further training for teachers.

In the light of modern developments in the mathematical field, the introduction of applied mathematics is envisaged.

No experimental work on the teaching of mathematics is at present being done, but it is hoped to undertake such work in the near future.

NETHERLANDS

From the reply sent by the University of Utrecht Institute of Education

Place of Mathematics in Curriculum

Mathematics is taught in all years of the upper primary schools (covering the first cycle of secondary education, pupils from 12 to 16 years of age), in all years of the modern secondary schools (pupils from 12 to 17 or 18 years of age) except the last two years of the modern " A " section (languages), and in all years of the grammar schools (pupils 12 to 18 years of age).

It forms one of the options at the upper primary schools, and is a compulsory subject in the other schools.

The following table shows the number of hours a week allocated to mathematics :

	1st yr.	2nd yr.	3rd yr.	4th yr.	5th yr.	6th yr.
Upper Primary Schools :						
Stream A	3	3	0/6	0/3	—	—
Stream B	3	3	3	4	—	—
Modern Secondary Schools :						
Modern section A (languages) .	5	5	5	0	0	—
Modern section B (science) . .	5	5	5	5	5	—
Classical section A (languages).	5	4	2	2	2	2
Classical section B (science) . .	5	4	2	2	5	5
Grammar Schools :						
Section A (languages)	5	4	3	2	2	2
Section B (science)	5	4	3	2	5	5
Lycées :						
Modern section A	5	4	5	0	0	—
Modern section B	5	4	5	6	5	—

Syllabuses

Upper Primary Schools. School leaving examination syllabus. Algebra : fundamental operations on numbers and algebraical expressions ; operations on powers with letters as indices ; involution ; resolution into factors ; highest common denominator and lowest common multiple ; a system of linear equations involving not more than three unknowns ; simple operations on radicals ; solution of simple equations of the second degree ; in addition, for section B only, properties of the roots of second degree equations, fractional and negative indices, logarithms, arithmetical and geometrical progressions, coordinates. Geometry : properties of segments of straight lines and of angles ; properties of triangles ; congruence of triangles ; construction of segments of straight lines, angles and triangles ; properties of parellelograms and trapeziums (constructions) ; ratios of

segments of a straight line ; similarity of triangles ; extension of the theorem of Pythagoras to any triangle (applications) ; construction of the fourth and the third proportional of the segments of given straight lines ; principal lines of a triangle ; area of a triangle, of a parallelogram, of a trapezium, and of a quadrilateral with diagonals at right angles ; simple properties of a circle ; construction of tangents drawn from a point to a circle, and of the geometric mean ; in addition, for section B only, construction of tangents common to two circles, simple loci ; circumscribed, inscribed and escribed circles and their simple applications ; inscribed and circumscribed quadrilaterals, properties and construction of some regular polygons, perimeter and area of a circle, elementary trigonometry.

Modern Secondary Schools. Modern sections A and B. 1st year. Arithmetic and algebra : natural numbers and fundamental operations on these numbers ; extension of the notion of number ; resolution into factors (the highest common factor and the lowest common multiple) ; mental arithmetic ; fundamental operations on entire polynomials ; principal identities ; solution of the equation of the first degree involving one unknown. Geometry : elements of plane geometry as far as the ratios of the segments of a straight line ; simple properties of a circle. — *2nd year.* Arithmetic and algebra : extractions of the roots ; simple approximations ; direct and inverse proportion ; rectangular coordinates (functions, graphical representation) ; arithmetical progression ; extraction of the root of an expression containing one term and of a binomial of the form $a \pm \sqrt{b}$; rationalization of the denominator of a fraction with the denominator containing not more than two square roots. Geometry : ratios of segments of a straight line ; homology ; similarity ; properties of right-angled triangles ; calculations associated with a right-angled triangle and any other triangle ; notion and simple properties of trigonometrical ratios, sine, cosine and tangent, with applications to geometry ; use of tables of trigonometrical ratios ; areas (simple constructions). — *3rd year.* Arithmetic and algebra : fractional indices and negative indices ; logarithms ; use of logarithmic tables to four decimal places ; arithmetical and geometrical progressions ; the notion of limits ; the function $y = ax^2 + bx + c$; second degree equations. Measurement of angles and trigonometry : trigonometrical ratios of one angle only ; graphical representation of functions of sine, cosine, tangent and cotangent ; use of tables ; application of trigonometry to right-angled triangles. Geometry : study of the circle (cont.) ; centres of homology of two circles ; ratios of the segments of the seccant of a circle ; inscribed, circumscribed and escribed circles ; area and perimeter of a circle ; simple properties of regular polygons. — *4th and 5th year* (modern section B only). Arithmetic and algebra : irrational equations and equations reducible to the second degree ; division of a polynomial in x by $(x - a)$; value of the remainder ; elementary infinitesimal calculus ; application to graphical representation and to problems of mechanics and physics ; functions $y = \dfrac{ax + b}{px + q}$ and $y = \dfrac{ax^2 + bx + c}{px^2 + qx + z}$; revision and extension of the notion of numbers ; general revision. Angular measurements and trigonometry : extension of the notions of angular measurements and trigonometry ; application to any triangle ; the radian as a measure of angles ; simple trigonometrical equations ; the function of $y = a \sin x + b \cos x$; equation

of $a \sin x + b \cos x = c$; general revision. Geometry (4th year) : simple geometrical properties of a parabola, of an ellipse, and of a hyperbola ; revision of geometry with applications of trigonometry ; solid geometry up to calculation of volumes (excluding the trihedral angle) ; plane sections and projections (revolution of the plane of projection) ; introduction to descriptive geometry. Geometry (5th year) : simple calculations of volumes (prism, pyramid, cylinder, cone, sphere, simple parts of a sphere) ; elements of spherical geometry ; regular polyhedrons ; plane sections of a cone ; development of a conic section ; descriptive geometry ; plane sections of bodies bounded by planes, a cone, a cylinder, and a sphere in a simple position.

Grammar Schools. 1st, 2nd, 3rd and 4th year. Algebra : fundamental operations with whole numbers, fractions and quantities ; divisibility ; ratio and proportion ; first degree equations in one or more unknowns ; radicals ; fractional indices and negative indices ; equations of the second degree ; logarithms ; graphical representation. Geometry : plane geometry ; introduction to trigonometry with applications to a triangle. — *5th and 6th year (section A).* Algebra : second degree equations (cont.) ; revision. Geometry : solid geometry ; revision of plane geometry. — *5th and 6th year (section B).* Algebra : as for section A, plus arithmetical and geometrical progressions, logarithms, elementary infinitesimal calculus, applications to problems. Geometry : as for section A, plus plane trigonometry, analytical geometry (including conic sections).

Methods of Teaching Mathematics

The method generally used for the teaching of mathematics is that of Euclid, adapted to the children's level of understanding. Some schools use the intuitive method at the beginning of the work in geometry. The only transformation dealt with in all schools is homologous transformation.

Descriptive geometry is taught in modern secondary schools, and analytical geometry in classical secondary schools. This differentiation will doubtless be abolished at some time in the future.

There are no official textbooks ; textbooks are not controlled. Each teacher is free to choose the textbook he prefers.

The methodology of mathematics is taught in five universities, but the professors concerned have published nothing on the subject. There are a number of books on the psychology of teaching in general, but none specifically relating to mathematics.

Teachers

Upper primary teachers are trained in normal schools. In order to teach mathematics they are required to hold the primary teacher's certificate in mathematics ; the examination syllabus for this certificate covers plane and solid geometry, algebra, and trigonometry, and emphasis is laid on ability to solve problems.

Half the secondary teachers of mathematics have had an academic training, and the remainder hold the secondary teacher's certificate, obtainable at the university or through special courses. There is no higher

normal school or seminar for training secondary teachers. Candidates for the secondary teacher's certificate are required to undergo examination before an official examining board which is independent of the university. At the present time 16% of the secondary teachers hold a certificate which does not fully qualify them. The lack of qualified secondary teachers is a matter of concern to the government.

Regulations concerning the professional training of secondary teachers were instituted for the first time on 1st September, 1955. In addition to their academic training, secondary teachers will henceforth be required to follow university courses in pedagogy, adolescent psychology, and general and special methodology, and to complete a period of teaching practice, lasting not more than six months, in a secondary school.

Miscellaneous

Since the end of the war the following endeavours have been made to improve the teaching of mathematics :

(a) In 1949 a committee of the association of mathematics teachers in modern secondary schools "WIMECOS" recommended the inclusion of infinitesimal calculus in the written part of the school leaving examination. Their aim was to bring the school leaving examination syllabus of 1929 into line with the 1937 curriculum ; the latter included subject-matter (the geometry of conic sections, spherical geometry, and infinitesimal calculus, for example) which did not appear in the 1929 syllabus.

(b) In a report published in 1950, a committee of the association of mathematics teachers in classical secondary schools "LIWENAGEL" recommended the inclusion of infinitesimal calculus in the baccalaureate examination syllabus for section B, and of optional matter in the mathematics syllabus for section A.

(c) In a report published in 1952 a Dutch sub-committee of the New Education Fellowship *(Wiskunde-werkgroep van de W.V.O.)* recommended that the teaching of mathematics be simplified by the removal of all useless subject-matter, and that the advanced syllabuses (infinitesimal calculus, statistics) be brought up-to-date.

(d) In a report published in 1955, a committee of the association of mathematics teachers in modern secondary schools "WIMECOS" recommended the exclusion of descriptive geometry from the modern secondary school baccalaureate, and its replacement by analytical geometry and statistics.

Mention should also be made of the research done in this connection at the University of Utrecht Institute of Education. The history of mathematics, for example, has been taught in the literary sections of a few classical secondary schools, and mathematics integrated with the programme of these sections. A book has been published on the history of mathematics, called *Van Ahmes tot Euclides*. Since 1951, moreover, statistics has been taught as an experiment in the literary sections of some classical secondary schools ; a report is shortly to be published on this experiment. The findings of an inquiry on the overburdening of the mathematics syllabuses were published in *Acta Paedagogica Ultrajectina*, No. V, 1953. Research is also being carried on by the Public Welfare Society in Amsterdam.

NEW ZEALAND

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

In New Zealand, general secondary schools are called " post-primary schools ". There is no clear demarcation between them and technical schools ; all post-primary schools prepare pupils for the School Certificate and University Entrance Examinations.

In the 1st, the 2nd, and the 3rd year (pupils 13 to 15) of post-primary education, all pupils must take at least elementary (" core ") mathematics ; pupils of or above average ability in mathematics generally study formal mathematics as a subject for the School Certificate Examination which comes at the end of this period, and they may also study applied mechanics for that examination.

In the 4th year (pupils of 16 plus) of post-primary education, pupils who have passed the School Certificate Examination may choose to take mathematics as a subject for the University Entrance Examination held at the close of that year. Usually only pupils above average ability in mathematics elect to do this. Mechanics may also be chosen as a subject for that examination.

In the 5th year (pupils of 17 plus) of post-primary education, pupils who have passed the University Entrance Examination may choose mathematics as a subject for the University Entrance Scholarship Examination. Pupils with special mathematical ability may also take additional mathematics for this examination.

The time allocation for mathematics varies considerably. In the first three years, two or three forty-minute periods a week (out of a total time-table of thirty-five periods) are allocated to elementary (" core ") mathematics, and six or seven periods to formal mathematis. In the 4th and the 5th year seven periods a week are allocated to mathematics.

Elementary (" core ") mathematics is mainly arithmetic with some algebra and geometry in mensuration form. Formal mathematics (taken for the School Certificate Examination) consists of arithmetic, algebra, geometry, and trigonometry. Mathematics for the University Entrance Examination is as for the School Certificate Examination, but to a much higher standard. Mathematics for the University Entrance Scholarship Examination is at a still higher standard and contains work on the calculus and analytical geometry ; the additional mathematics for this examination demands a fuller treatment of the calculus and analytical geometry, and, in addition, applied mechanics.

Formal mathematics (for the School Certificate Examination) is often optional, especially in girls' schools and in girls' sections of coeducational schools ; the option is not usually introduced into boys' schools until the 2nd and often the 3rd year. If formal mathematics is not taken, then at least the syllabus in elementary (" core ") mathematics must be covered. In junior classes, mathematics, English, social studies, and science are

the subjects usually used as a basis for promotion. Mathematics is one of over thirty subjects for the School Certificate Examination.

Aims of Mathematics Teaching

The aims of mathematics teaching are to equip every pupil with sufficient skills to meet the demands of everyday life, to enable pupils to pass qualifying examinations, to equip pupils to carry out competently further studies leading to a variety of professions, and to give them a broad cultural background.

Syllabuses

Elementary mathematics (the minimum requirement in the first three years) consists essentially of the practical aspects of arithmetic, with some practical algebra and geometry. Formal mathematics (one of the options for the School Certificate Examination) comprises arithmetic (mensuration, percentages, profit and loss), algebra (solution of equations and problems, graphs), geometry (plane geometry, including the circle), and trigonometry (heights and distances, problems for angles up to 180°).

Mathematics for the University Entrance Examination comprises work similar to that for the School Certificate Examination, but carried to a much higher standard, including harder equations and graphs in algebra, three-dimensional geometry, and a fuller treatment of angles and triangles in trigonometry.

Mathematics for the University Entrance Scholarship is at a still higher standard, and contains work, as already noted, on the calculus and analytical geometry. The additional mathematics for this examination demands a fuller treatment of the calculus (integration and differentiation of trigonometrical functions), geometry (ellipse, parabola), and algebra (cubic equations).

Mechanics, whether for the School Certificate or the University Entrance Examination, consists of the respective levels of statics, dynamics, and hydrostatics.

The history of mathematics is taught incidentally. The principles of axioms are included in the formal mathematics, but not in elementary (" core ") mathematics.

Methods of Teaching Mathematics

No official instructions are issued concerning methods of teaching mathematics at post-primary level, but teachers and heads of departments are assisted in this connection by refresher courses, inspectors, and bulletins. The main principles suggested to teachers are as follows : to use short mental work at the beginning of each lesson ; to work out on the blackboard only those homework problems incorrectly done by a large proportion of the class ; to make the teaching part of the lesson short, and not to include too much new work ; to get the pupils working on problems as early as possible in the period ; to give as much individual help as possible ; to insist on high standards of neatness and setting out ; to make sure sufficient work is set to keep the better pupils working to capacity.

No official instructions are issued concerning methods for use in the transition stage from primary to secondary education. Correlation between mathematics and other subjects is ensured through consultations between the heads of the various departments.

The teaching aids used for mathematics lessons include films, filmstrips, charts, models, and carefully selected books.

Practical work in mathematics is used extensively in the 1st and the 2nd year, especially with low-ability groups. Applied mathematics, as already noted, is sometimes included in the 3rd, the 4th, and the 5th year, but only for special pupils, mainly in the engineering courses.

There is an official list of authorized textbooks. A considerable amount of freedom, however, is permitted to teachers in this connection, especially in the senior classes. New textbooks are brought to the notice of teachers by means of reviews in the official gazette. During their visits to schools, moreover, inspectors usually discuss textbooks with teachers.

Teachers

No special training is required of post-primary teachers of mathematics, but it is considered desirable that they should have a background of university training in the subject, and have spent one post-graduate year in a teacher training college.

Great difficulty is experienced in recruiting post-primary teachers of mathematics. The reasons for this are the keen competition for the services of mathematics graduates, and the falling off in the number of graduates in this subject, relative to that of graduates in other subjects. One measure taken to remedy the difficulty is the award of an increasing number of bursaries for post-primary teachers; it is difficult, however, to obtain sufficient suitable applicants to take up all the bursaries offered; the whole question of bursaries is kept constantly under review.

There are a number of opportunities for further training. Refresher courses, organized by the teachers' refresher course committee (and financed by the Department of Education) or the university colleges, are held frequently. Several districts have formed mathematics associations which provide specialized instruction and encourage the interchange of ideas. Travel scholarships, limited in number and not restricted to mathemathics teachers, are available.

Miscellaneous

The syllabuses for the various examinations, including those for mathematics, have been changed recently, and will therefore probably continue to be in force for a considerable period.

Modern developments in the field of mathematics have as yet made no impact on New Zealand schools.

The Department of Education has been responsible for the compilation, but not the publication, of a new textbook for elementary ("core") mathematics. Apart from this, no work of an experimental nature has been carried out.

The handbooks on the teaching of mathematics used in New Zealand are all overseas publications.

NICARAGUA

From the reply sent by the Ministry of Public Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout secondary education, the duration of which is five years, the age range of the pupils being from 13 to 18 years. Five hours a week (16.66% of the total time-table) is allotted to the subject in the 1st, the 2nd, and the 3rd year, and three hours (10%) in the 4th and the 5th year.

Mathematics is designated *aritmética razonada y comercial* in the 1st year, *álgebra* in the 2nd year, *geometría plana y del espacio* in the 3rd year, *trigonometría, nociones de topografía* in the 4th year, and *revision de matemáticas* in the 5th year.

No greater importance is in general attached to mathematics than to other subjects in the secondary curriculum. It is laid down in official regulations that one member of the baccalaureate examination board must be a mathematics specialist.

Aims of Mathematics Teaching

In common with the other subjects in the secondary curriculum, mathematics is expected to make its contribution to realizing the general aims of secondary education, which are formulated as follows in article 10 of the general regulations on secondary education *(Reglamento general de segunda Enseñanza)* : " The aim of the national and independent secondary schools is to give Nicaraguan pupils a general education based on the all-round development of their aptitudes, and preparation for higher studies." No specific reference is made to the aims of mathematics teaching in official texts.

Syllabuses

1st year. Arabic and Roman figures, the four fundamental operations, raising to a power and extraction of the roots of whole numbers, decimals, and fractions, operations on compound numbers ; the decimal system of measurements, and comparison of it with other systems ; ratios and proportions, proportional parts, the rule of three, percentages, interest, discount, proportional parts, averages, mixtures, alloys ; principal commercial operations. — *2nd year.* Algebraical expressions and their numerical value, operations on algebraical numbers, divisibility and the highest common factor ; equations of the first degree with one unknown, systems of equations with several unknowns ; functions, graphical representation ;

second-degree equations, irrational and biquadratic equations, arithmetical and geometrical progressions, logarithms, compound interest. — *3rd year.* Study of the elements of geometry, angles, triangles, polygons, circles, similar figures, relations in the elements of the triangle and the circle, power of a point in relation to a circle, areas and volumes. — *4th year.* Functions and trigonometrical lines, sine, cosine, tangent, cotangent ; solution of triangles, operations involving the use of logarithms ; notions of surveying and the required instruments, calculation of area and topographical plans. — *5th year.* Revision of the work done in previous years, problems and application exercises.

The official syllabuses do not cover the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

The education authorities do not stipulate the use of any given method of teaching mathematics. The teachers generally adopt a theoretical and rational approach, make almost exclusive use of analysis, and do not give any special attention to practical conclusions. No steps have been officially taken to facilitate the transition from primary to secondary education.

The materials used for mathematics lessons include rulers, set-squares, geometrical figures and solids, and instruments for measuring and calculating.

The teachers ensure the coordination of mathematics with other subjects in the curriculum, to the extent that they teach the basic mathematical knowledge before proceeding to work in physics, chemistry or cosmography, for a proper understanding of which such knowledge is necessary.

Questions affording a practical application of mathematical knowledge (commerce, the calculation of interest, topographical plans, etc.) are also dealt with, but mainly as preparation for higher studies.

Activity methods are not generally used for the teaching of mathematics.

The Ministry of Education does not require the use of any given textbooks, but may specially recommend certain textbooks.

Teachers

There is no special course for the training of secondary teachers. Mathematics is thus taught at secondary level by holders of the baccalaureate certificate with several years' teaching experience, or by persons from outside the teaching profession who possess recognised qualifications. In view of the absence of adequate training, there is is a lack of teachers with a knowledge of the methodology of mathematics teaching.

Secondary mathematics teachers are occasionally asked to attend general meetings to consider and discuss the teaching of their subject. Several students holding scholarships are at present pursuing specialist studies in mathematics abroad.

Miscellaneous

The Ministry of Education plans to make a careful study of the mathematics syllabuses now in force, with a view to bringing the teaching of the subject more into line with the progress made in the mathematical field, with modern teaching ideas, and with the interests of the pupils. The establishment of a higher institute of education for training secondary teachers, including those for mathematics, is also envisaged.

NORWAY

From the reply sent by the Royal Ministry of Church and Education

Place of Mathematics in Curriculum

Secondary education comprises modern secondary schools *(realskole)* with a three-year course, the age range of the pupils being from 14 to 17 years, and grammar schools *(gymnasium)* with a five-year course, the age range of the pupils being from 14 to 19 years.

Mathematics is taught as a compulsory subject in all years of the modern secondary schools. Five hours a week (16.12% of the total time-table) is allotted to the subject in the 1st and the 2nd year, and four hours a week (12.5% of the total time-table) in the 3rd year. Mathematics is designated arithmetic, algebra, geometry, and applied arithmetic in the 1st and the 2nd year, and " social mathematics " in the 3rd year.

Mathematics is also taught as a compulsory subject in the grammar schools, in all five years of the course in the mathematics and the natural science section, and in the first four years in the linguistic sections. The time allotted to the subject in the 1st and the 2nd year is the same as in the modern secondary schools, and as follows in the 3rd, the 4th and the 5th year :

	3rd year		4th year		5th year	
	Hours	%	Hours	%	Hours	%
Mathematics Section . .	6	20	6	20	8	28.59
Natural Science Section .	4	12.5	5	16.12	5	16.12
Linguistic Section . . .	4	12.5	5	16.12	—	—

In the grammar schools mathematics is designated, in the 1st and the 2nd year, the same as in the corresponding years of the modern secondary schools, and as follows in the 3rd, the 4th and the 5th year : *mathematics section* : algebra and plane geometry in the 3rd year, trigonometry, solid geometry, algebra, analytical geometry, and projective drawing in the 4th year, and algebra, analytical geometry, infinitesimal (differential and integral) calculus in the 5th year ; *natural science section :* this being a new section, its syllabuses have not yet appeared in their final form ; *linguistic sections :* algebra, plane geometry, applied arithmetic, and elementary trigonometry in the 3rd and the 4th year.

In promotion examinations the same importance is attached to mathematics as to the other subjects in which there is a written test (Norwegian, German, English, physics, Latin). In the two secondary school leaving examinations the mark obtained for the oral and the written papers in mathematics is multiplied by two ; the only subjects ranking higher than this are English (in the English section) and Latin (in the Latin section), the respective total marks for which are multiplied by three.

Aims of Mathematics Teaching

The aims of mathematics teaching in the modern secondary schools are essentially practical ones, as follows : to impart insight into, and skill in solving, arithmetical problems from practical life ; to give insight into elementary algebra, and impart understanding and command of the mathematical language of symbols ; in general to impart basic knowledge of arithmetic, geometry, and mensuration.

The aims of mathematics teaching in the 1st and the 2nd years of the grammar schools are the same as those in the corresponding years of the modern secondary schools. In the 3rd, the 4th and the 5th year they are, in the science section, to impart a more complete understanding of algebra, geometry, trigonometry, and infinitesimal calculus, and thus to prepare the pupils for practical and technical applications and for the study of theoretical problems and research, and, in the linguistic sections, to impart ability to solve practical problems and an understanding of economic relationships.

Syllabuses

Modern-secondary and grammar schools. 1st and 2nd years. Arithmetic and algebra : the four elementary operations, fractions, powers, roots, approximate numbers, addition and reduction, proportions, equations of the first degree with one and two unknowns. Geometry : an introduction to plane geometry (theorems, constructions, the measurement of angles, straight lines, areas) ; similar triangles. Applied arithmetic : commercial arithmetic (interest, foreign exchange, bills of exchange, etc.) ; the rule of three, mixtures and alloys, weight, specific weight, etc.

Modern secondary schools. 3rd year. Bookkeeping and social mathematics (calculation in relation to production, finance, insurance, taxation, etc., and graphical representation).

Grammar schools. Mathematics and natural science sections. 3rd year. Algebra : equations of the second degree, systems of equations of the first and the second degree, graphical representation of functions, real and approximate numbers, the reduction of root expressions, logarithms. Geometry : similar figures ; methods of geometrical constructions and calculations, with discussion ; homology, regular polygons. — *4th year.* Trigonometry : trigonometrical functions with their logarithms, formulas for the measurement of angles, area of triangles, trigonometrical equations. Geometry : elements of solid geometry. Algebra : rules of derivation, maxima and minima, the discussion of curves, some analytical geometry (straight lines and the circle). — *5th year.* Algebra : integral calculus, the relations between roots and coefficients in equations of any degree, arithmetical and geometrical series, annuities. Analytical geometry : parabolas, ellipses, hyperbolas, conic sections, etc.

Grammar schools. Linguistic sections. 3rd and 4th years. Algebra : problems, equations, graphical representation of functions, equations of the first and the second degree, powers with rational exponents, logarithms, arithmetical and geometrical series, compound interest and annuities.

Plane geometry: similar figures, constructions. Applied arithmetic: calculations relating to industry, insurance, taxation, etc.

The mathematics syllabuses make no reference to the history of mathematics. The textbooks, however, contain historical surveys, annotations and footnotes. The mathematics syllabuses make no reference to the principles of axioms.

Methods of Teaching Mathematics

No official instructions concerning methods of teaching mathematics are issued, but all secondary teachers of mathematics have had training in methods, including a period of practice teaching under competent supervision.

In the teaching of algebra the ordinary rules for calculating with numbers are converted into formulas right from the beginning, and proofs are only given where necessary for understanding. The early stages of the geometry course are built on previous experience and intuition, and ordinary methods of proof are used only when the truth of the theorems is not immediately obvious. In class teaching, each individual pupil may make his contribution, and new topics are as a rule introduced by the heuristic or genetic method.

No special instructions have been issued bearing on teaching methods to be used at the transfer stage from primary to secondary education.

With regard to the correlation between mathematics and other subjects, it is to be noted that topics requiring mathematical treatment in subjects such as physics and economics, for example, are taken up only after a sufficient mathematical foundation has been laid. Other subjects, moreover, are made use of in mathematics for the purposes of illustrations and problems.

The materials for mathematics lessons include, for the pupils, sets of drawing instruments, set-squares, protractors, and slide-rules, and, for the teacher, materials for use in connection with the blackboard, and a collection of models of geometrical solids. Films and slides are not generally used in the teaching of mathematics.

The mathematics course is too short to allow a great deal of time to be devoted to practical exercises, but direct measuring on figures is often undertaken in order to test a new theorem or to check calculations.

On the recommendation of the Council of Education, all textbooks must be approved by the Royal Ministry of Church and Education. The textbook employed by each class must be chosen by the teaching staff. The authors of textbooks are free as to the order in which the various topics are presented, but must cover all the topics mentioned in the syllabuses.

Teachers

Secondary teachers of mathematics are required to hold a university degree with mathematics as major or minor subject, to have completed the prescribed period at a teacher training school, and to have passed the examination for the award of the title *cand. real.* or *cand. mag.*

Some difficulty is experienced at the present time in recruiting university trained teachers of mathematics, and the shortage is expected to increase in the next few years. The shortage is due to the fact that a greater proportion than before of mathematically trained persons have gone into research, industry, and life insurance companies, because such openings offer better prospects than teaching. No measures have so far been taken to remedy the shortage.

Secondary teachers have formed associations for their special subjects, which hold regular meetings for lectures and discussions on topics concerning their work, and arrange vacation courses. The schools and many individual teachers of mathematics subscribe to the *Nordisk Matematisk Tidsskrift* (nordic mathematics journal), which contains articles on mathematics and on the teaching of the subject. Travel scholarships are granted by the government, the municipalities, and some foundations.

Miscellaneous

New mathematics syllabuses were introduced in 1950, and show the influence of modern developments in the mathematical field. They include, for example, infinitesimal calculus as a compulsory item for the mathematics section, and "social mathematics" for the modern secondary schools and the linguistic sections of the grammar schools.

Teachers are free to adopt teaching methods differing from the ones traditionally employed, provided they apply for, and obtain, permission to do so.

PAKISTAN

The educational system of Pakistan is decentralized, and the various provinces have sent separate replies to the questionnaire of the present inquiry. It has not proved possible to cover all the data received, and the report below therefore gives, as an example, details of the teaching of mathematics in East Bengal.

East Bengal

From the reply transmitted by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the secondary course, which extends from the 6th to the 10th class, the age range of the pupils being from 11 to 15 years. The subject is designated " arithmetic and geometry " in the 1st year (class VI), and " arithmetic, algebra and geometry ", in the remaining years.

Six forty-minute lessons a week are allotted to mathematics, which represents approximately 15.4% of the total weekly time-table of thirty-nine forty-minute lessons. Optional additional teaching in mathematics is given in the last two years.

Mathematics ranks as a major subject in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The aims of mathematics teaching at secondary level are as follows : to develop the pupils' understanding for the need of mathematics to solve individual and social problems ; to develop their ability to think in mathematical terms ; to inculcate in them certain attitudes, habits, skills and interests, and ideals relating to democratic living ; to lead the pupils to visualize mathematics as a means to self-realization, economic efficiency, civic responsibility, and the interpretation of culture.

Syllabuses

1st year (class VI). Arithmetic : multiplication, division ; calculations involving money, weights and measures ; vulgar and decimal fractions. Geometry : ratios, angles, the sum of the angles of a triangle and a polygon, congruency of triangles, the bisection of angles and straight lines, construction of perpendiculars. — *2nd year* (class VII). Arithmetic : revision

of the previous year's work ; exercises on ratios and proportions. Algebra : elementary algebraical calculations, algebraical expressions, negative numbers, the four operations, linear equations. Geometry : symmetry about a point or a line, the isosceles triangle, parallelograms, rectangles, squares, trigonometrical ratios, similarity of triangles ; the construction of parallels, angles, triangles and simple quadrilaterals ; bisection of angles and of straight lines. — *3rd year* (class VIII). Arithmetic : revision of the previous year's work ; the metric units, the calendar, percentages, interest, profit and loss. Algebra : multiplication, factors, trinomials, fractions, identities, linear fractions involving fractions, equations of the second degree, equations with two unknowns. Geometry : revision of the previous year's work ; properties of the areas of rectangles, parallelograms, triangles and trapezia ; applications of algebra to geometry (coordinates). — *4th and 5th years* (classes IX and X). Arithmetic : the four operations, vulgar and decimal fractions, reductions, extraction of square root, proportions, interest, discount, stocks and shares.

The secondary mathematics syllabuses do not include the history of mathematics. Teachers refer to the principles of axioms in the course of the work, but do not give any systematic introduction to them.

Methods of Teaching Mathematics

There are official recommendations concerning the methods of teaching mathematics at secondary level. The analytic method is commonly used in respect of beginners, and the synthetic method when the pupils have acquired a certain proficiency.

At the transition stage between primary and secondary education, teachers undertake a recapitulation of the entire primary course in mathematics.

The teaching aids used include films and slides, etc.

Mathematics is taught at the secondary level as a principal and separate subject having only incidental correlation with other subjects such as geography, physics, astronomy, etc.

Some attention is given to practical applications, involving exercises in measurement and work in mechanics, trigonometry, etc. The use of the analytic method, moreover, involves measurements, handicrafts, surveying, etc.

The textbooks selected for the various classes must be ones approved by the textbook committee.

Teachers

Secondary teachers of mathematics are required to be science and mathematics graduates with special training in the teaching of mathematics. No difficulty is experienced in recruiting such teachers.

Serving teachers have a number of means at their disposal for keeping up-to-date in their subject, including refresher courses, teacher exchanges with countries abroad, the work of the association of mathematics teachers, and the educational journals.

Miscellaneous

The entire educational system is being recast, and it is as yet impossible to say what changes will be made in the teaching of mathematics. Great importance, at all events, is attached to mathematics at secondary level, as the subject forms the basis of the whole system of higher scientific education, and the syllabuses are modified from time to time in the light of modern developments in the mathematical field.

With a view to evolving better methods of teaching, experiments and special studies are undertaken from time to time in the teacher training schools, and groups of teacher-trainees are asked to work upon different methods during their practice teaching, the results thus obtained being carefully assessed.

PANAMA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Secondary education comprises a three-year cycle of general education (pupils from 12 to 16 years of age), and a three-year cycle of preparation for the baccalaureate in science or arts (pupils from 16 to 20 years of age).

Mathematics is taught throughout the secondary course. The subject is compulsory in the 1st and the 2nd year, and optional thereafter. In the curriculum it is designated *matemáticas*.

Four hours a week is devoted to mathematics in each of the three years of the first cycle, this allocation representing 12.5%, 13.7% and 12.12% respectively of the total time-table. Five hours a week is allotted to mathematics in each of the three years of the second cycle.

The same importance is attached to mathematics as to other subjects.

Aims of Mathematics Teaching

The aims assigned to the teaching of mathematics are general educational aims, and specific aims.

The general educational aims of mathematics are to help the pupils to realize the importance of the mathematical aspect of human activities, to awaken their interest in the mathematical field of knowledge and to show them its correlation with related subjects, to give them opportunities for experiments and activities contributing to the formation and strengthening of personality, to impart habits of thought and action (reflection, analysis, clarity, perseverance, attention, self-criticism, etc.), and to develop logical reasoning.

The specific aims of mathematics teaching have been defined by the education authorities as being to impart (a) the fundamental mathematical ideas, (b) the notions and skills necessary for the use of mathematical symbols, the evaluation of quantitative data, and the application of these to social phenomena, and (c) the capacity to use algebra as a means of expressing concrete situations in a generalized and abstract form.

Syllabuses

1st year. Arithmetic : numeration, the fundamental operations, vulgar and decimal fractions, lines and angles. — *2nd year.* Percentages and their applications, a study of graphs as a means of representing mathematical situations. Geometry : triangles, quadrilaterals. — *3rd year.* Algebra : operations on polynomials, prime products, the highest common factor, solution of problems by means of equations. Geometry : a study of parallel lines, a study of similar triangles and their application, introduction to deductive geometry, a study of parallelograms and of the circle. — *4th year.* Algebra : the highest common factor, algebraical fractions, ratios and

proportions, systems of equations of the first degree, powers and roots. Geometry : a study of the circle and similar polygons. — *5th year.* Algebra : the theory of numbers, logarithms, equations of the second degree with one unknown, equations of the second degree with two variables. Trigonometry : trigonometrical ratios ; trigonometrical relations in the triangle, functions, elements of solid geometry. — *6th year.* Algebra : progressions, the binomial theorem of Newton, ratio of functions, inequalities, rational equations. Analytical geometry : coordinates, straight lines, conic sections. Trigonometry : trigonometrical ratios and functions, solution of triangles, properties and formulas.

The history of mathematics is not given separate treatment. Teachers make historical references whenever the need to do so arises.

Methods of Teaching Mathematics

Suggestions have been drawn up concerning suitable methods of teaching mathematics at secondary level. Teachers are recommended to be fully conversant, not only with their subject matter, but also with modern methods of teaching it, to understand their pupils and to take account of their individual differences, to awaken the interest and encourage the active participation of their pupils, to make all possible use of projects and directed work, to give emphasis to the fundamental notions, to evaluate frequently the results of their teaching and to be familiar with the methods of evaluating school work, to take account of the conditions under which homework has been done and to mark it individually, and to ensure that their pupils work with understanding and not mechanically.

The teaching materials used for mathematics lessons include filmstrips, films, geometrical solids, and various materials in wood and cardboard.

The correlation of mathematics with other subjects is ensured as and when the opportunity to do so presents itself, and not methodically in advance. The syllabuses include certain practical exercises serving to maintain the pupils' interest in the theoretical notions ; this is especially the case in regard to the geometrical problems and those relating to the pupils' environment.

The education authorities do not prescribe the use of any given mathematics textbooks, but nevertheless exercise a certain control over all the materials used by both teachers and pupils.

Teachers

Most secondary teachers of mathematics hold the secondary teacher's diploma required of first-category teachers, with mathematics and physics as special subjects. No difficulty is experienced in recruiting such teachers.

The secondary teachers of mathematics were convened in two seminars (May, 1952, and February, 1955) to draw up new syllabuses and to discuss the results obtained.

Miscellaneous

Modifications were made in the new syllabuses in 1953, involving more the distribution of subject matter than its content. These syllabuses are now being tried out.

PERU

From the reply sent by the Ministry of Public Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the five years of secondary education. Four hours a week is allocated to the subject, i.e., 13.3% of the total time allocated to lessons. The age range of secondary pupils is usually from 12 to 16, or to 18, 19 or 20 in the case of the considerable number of pupils who begin their secondary studies later than at 12 years of age.

Mathematics is officially designated *aritmética* in the 1st year, *álgebra* in the 2nd and the 3rd year, *geometría plana* in the 4th year, and *geometría del espacio y nociones de trigonometría* in the 5th year.

The children's work in mathematics is evaluated in the same way as other subjects ; this also holds for promotion and school leaving examinations.

Aims of Mathematics Teaching

It is officially laid down that the aim of teaching mathematics in the 1st year is the essentially practical one of completing and consolidating knowledge gained at primary level, and of preparation for more advanced studies. In the following years the aim is to teach pupils to solve the problems that most commonly arise, and to develop their reasoning powers and their aptitude for science.

Syllabuses

1st year. Quantity, numbers, the four fundamental operations, raising to a power, prime numbers, highest common factor and lowest common multiple, fractions and operations with fractions, extraction of square root, the decimal systems of measurements, ratio and proportion, rule of three, interest, discount, proportional parts, mixtures. — *2nd year.* Introduction to algebra, algebraic expressions and operations, involution, common factors, algebraic fractions, first-degree equations with one unknown and systems of equations with several unknowns. — *3rd year.* Second-degree equations, graphical representation, arithmetical and geometrical progressions, logarithms and their applications. — *4th year.* Fundamental concepts of geometry, angles, polygons, the circle, similar figures, relations concerning the magnitudes of the elements of the triangle, inscribed and circumscribed polygons, areas. — *5th year.* Lines and planes in space (determination of a plane, relative position, etc.), diheóral angle, polyhedrons, projections, area and volume of polyhedrons, surface of revolution, round bodies, trigonometrical ratios, functions, sine and cosine.

The official mathematics syllabuses make no reference to the history of mathematics.

Methods of Teaching Mathematics

Broad indications are given in the suggestion to teachers contained in the official mathematics syllabuses as to ways in which the subject may be taught. During their training the specialist teachers of mathematics receive instruction in teaching methods: observation, demonstration through examples, analysis, synthesis.

No special methods are known for use during the transition from primary to secondary education, but teachers invariably do all in their power to facilitate it. They first find out how much their pupils have learned at primary level, and adapt their teaching accordingly.

All secondary schools are equipped with materials for teaching mathematics, especially materials for lessons on the decimal system of measurements, and geometrical figures. Where specific materials are lacking, teachers make use of pictures, drawings, diagrams, etc., according to the particular aspect of the subject with which they are dealing. No use is made of films or filmstrips.

While the knowledge acquired through mathematics teaching is of course applied to work in the physical, chemical and economic sciences, each of the subjects involved is taught quite independently.

Thanks to the suggestions contained in the official syllabuses and the training the teachers have undergone, mathematics is taught with a definitely practical bias, and the pupils are given practical exercises to do at frequent intervals. In order to ensure, moreover, that pupils participate fully in the lessons, in accordance with activity methods, they are normally required to keep a notebook *(carpeta de trabajo)* made up of their written work, the results of personal researches, and notes on books they have consulted.

Mathematics textbooks are examined and selected by specialists under the supervision of the secondary and higher education directorate in the Ministry of Public Education, and those which are approved may be adopted by secondary schools. In schools where no textbook is used, the notebook *(carpeta de trabajo)* the pupils are required to keep assumes greater importance.

Teachers

The teachers of mathematics receive their training either in the national institutes of education, along with other specialist teachers, or in the university faculties of education. In either case the training course consists of four years at higher level. In cases where a specialist teacher is not available, a civil engineer may be appointed to teach the subject. No difficulty is experienced in recruiting mathematics teachers.

Further training for teachers is not organized systematically, but every endeavour is nevertheless made to provide it, in the form of vacation courses and meetings of various kinds.

Miscellaneous

The mathematics syllabuses now in force were drawn up as recently as 1951. They take due account of modern developments in the mathematical field, and no modifications in them are at present envisaged.

No experimental research is conducted on the methodology of mathematics, but this topic is regularly considered and discussed by the teachers' committee and the headmaster of the larger schools.

PHILIPPINES

From the reply sent by the Department of Education

Place of Mathematics in Curriculum

There are three types of secondary school, each with a four-year course : general, agricultural, and trade secondary schools. The present inquiry concerns only the first of these three types, the general secondary schools, in which mathematics is taught in the 2nd and the 3rd year.

General mathematics is taught under that name as a required, one-unit subject throughout the 2nd year of the general secondary schools ; three and a third hours a week are allocated to it, constituting 12.5% of the total time-table. Advanced arithmetic is taught as a required half-unit subject for a semester in the 3rd year ; three and a third hours a week are allocated to it, constituting 12.5% of the total time-table.

Furthermore, advanced algebra, and geometry, are taught for one and two semesters respectively as optional subjects in the 3rd year ; to each of them three and a third hours a week are allocated, constituting 11% of the total time-table.

No special importance is attached to mathematics, in the general secondary schools, in comparison with other subjects.

Aims of Mathematics Teaching

The aims assigned to the teaching of mathematics are (a) to make mathematics more functional in the home, school, and community, thus enabling the pupils to solve problems encountered in everyday life, (b) to develop critical, quantitative, and logical thinking, (c) to develop greater appreciation of mathematics as a cultural heritage, and as a past and present factor in man's daily life and in human progress, and (d) to develop such desirable character traits as honesty, accuracy, perseverance, promptness, self-reliance, etc.

Syllabuses

Arithmetic and Algebra : numbers, ratio, proportion, fundamental operations, linear equations in one or more unknowns, special products and factoring, fractional equations, quadratic equations in one unknown and systems involving quadratic equations, powers and roots, logarithms, trigonometry of the right angle, statistics and graphs, practical problems concerning thrift in the home, the community and the nation, etc. Geometry : triangles, angles and parallel lines, quadrilaterals, polygons, circles, proportions, inequalities, similar polygons and regular polygons, etc.

The syllabuses also include some mention of the history of mathematics and the study of axioms.

Methods of Teaching Mathematics

Suggestions concerning the methods to be used in teaching mathematics have been issued by the education authorities. Teachers, however, are given complete freedom to modify the suggested methods and adapt them to local conditions. The methods employed generally comprise the following steps: exploration; presentation, with considerable use of the inductive and experimental approaches; directed study; testing, followed where necessary by remedial teaching, re-testing, and re-teaching to the point of learning.

As a means of adapting instruction to individual differences, moreover, the group method is used in all the classes in mathematics. Each class is divided into four or more groups, each group having a leader, who is usually the brightest member.

At the transfer stage from primary to secondary education, secondary mathematics teachers are instructed to strengthen the pupils' weak points in elementary mathematics, as determined at the beginning of the school year. Correlation between the teaching of mathematics and other subjects such as physics and economics, is established by including in mathematics lessons problems involving the other subjects.

The following teaching materials and auxiliary aids may possibly be used in mathematics teaching: films, filmstrips, slides, charts (graphs of national income and expenditure, for example), blackboard equipment (compasses, protractor, etc.).

With a view to making the teaching as practical as possible, the problems set are derived from everyday activities and transactions involving computational and reasoning skills. Formal teaching is preceded to some extent by practical exercises.

The textbooks used for mathematics are selected and approved by the textbooks board of the Department of Education. This board requires such textbooks to be functional and practical, and to lay stress on student interest, so that the pupils will be able to use the mathematical skills they acquire, in their present and future daily activities and transactions.

Teachers

Secondary teachers of mathematics are required to obtain thirty units in mathematics, and to hold the degree of Bachelor of Science in Education, with mathematics as major subject.

Some difficulty is being experienced in recruiting secondary mathematics teachers at the present time. Geometry and advanced algebra were made optional, which decreased the demand for mathematics teachers, and this in turn led to a considerable reduction in the number of teacher-trainees majoring in mathematics. No special steps appear to have been taken to overcome the difficulty.

Secondary teachers of mathematics are able to keep up-to-date in their knowledge through reading journals and books on mathematics and the teaching of the subject, conferences and workshops, reciprocal visits to each other's schools, study courses, and lectures by specialists.

Miscellaneous

Modifications are contemplated not only in the teaching of mathematics, but also in the teaching of other secondary school subjects. A workshop on " Improving the Secondary School Curriculum " was held at the University of the Philippines from 15th April to 6th May, 1955. The modifications envisaged by this workshop involve content materials and their distribution over the course, teaching methods, teacher training, and evaluation methods.

Modern developments in the mathematical field have not affected the teaching of the subject at secondary level.

Research into the teaching of mathematics includes in particular a study now being carried out at Manila on the effectiveness of homework in mathematics.

No books have been published recently on the teaching of mathematics at secondary level. The mathematics inspectors, however, from time to time publish articles in this connection in the educational journals.

POLAND

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

In Poland there is only one kind of general secondary school. It has a four-year course, covering classes VIII to XI, and the age range of the pupils is from 14 to 18. Mathematics is taught as a compulsory subject in all classes.

Five hours a week (14.7% of the total number of hours) is allocated to mathematics in the 1st year, four hours (10.3%) in the 2nd year, four hours (11.8%) in the 3rd year, and four hours (11.8%) in the 4th year.

Mathematics is designated as such in the secondary curriculum.

The following passage from the introduction to the secondary curriculum shows that more importance attaches to mathematics and the mother tongue than to other subjects: " Mathematics is very valuable for the general training of the mind and for acquiring a capacity to express ideas precisely and scientifically, and therefore, as an important factor in forming the mind and a scientific conception of the world, takes first place, together with the mother tongue, in the curriculum."

This high estimate of the value of mathematics finds expression in the regulations issued by the school authorities. To pass the promotion examination at the end of the 1st, the 2nd, and the 3rd year, pupils must get an average mark not below the " satisfactory " standard for the written and oral mathematics papers. This mark is granted by the educational council, and due account is taken of a pupil's work during the year. Written and oral mathematics papers are included in the school leaving certificate examination.

The special importance attached to mathematics finds expression also in requirements regarding further training for teachers of the subject, and in the organization of competitions at district and *voivod* level among pupils in the 1st and the 2nd year, and so-called " Olympic " competitions at national level among pupils in the 3rd and the 4th year.

Aims of Mathematics Teaching

The aims of teaching mathematics are classified in the introduction to the secondary curriculum under three heads: (a) acquisition of knowledge; (b) instructional aims; (c) educational aims.

This classification stems from the conception that mathematics forms part of the natural sciences, of the knowledge that shadows forth in the human mind the external world existing independently of human volition. Mathematics is expected to enable man to conquer nature, and for this reason the introduction to the secondary curriculum lays emphasis on the linking of theory and practice. The exercises the pupils are given

to do in class bear on subjects which, drawn from their surroundings, familiarize them with their country's present problems by introducing them as fully as possible to questions of production and the practical application of mathematics; this approach is of great importance in polytechnical education.

Syllabuses

1st year (class VIII). *Algebra:* systematic study of whole and fractional algebraic expressions, including resolution into factors; first-degree equations and systems of first-degree equations, including their graphical representation. *Geometry:* fundamental geometrical figures, parallel lines, symmetry and coincidence of figures, the parallelogram, the triangle, the circle, regular polygons, the method of using loci in the construction problems of geometry. — *2nd year* (class IX). Algebra: radicals, second-degree equations and systems of second-degree equations, graphical representation. Geometry: measurement of geometrical dimensions; circumference and area of a circle; construction of segments of a straight line according to a given algebraic formula. — *3rd year* (class X). Algebra: application of the concept of power to real indices, including the concept of the exponential function; logarithms, exponential and logarithmic equations, arithmetical progressions. Trigonometry: trigonometrical functions and their application to right-angled triangles. Geometry: stereometry, including parallel projection on a plane (points, lines and planes in space, surface and volume of polyhedrons). — *4th year* (class XI). Algebra: resolution of first-degree and second-degree inequalities, fractional inequalities. Trigonometry: fundamental trigonometric transformations, trigonometrical equations, relations among the elements of a triangle. Geometry: solids of revolution.

The mathematics syllabuses for general secondary schools include an introduction to the history of mathematics, showing the dependence of the evolution of mathematical concepts and symbols on the development of man's creative powers, and giving information about the great mathematicians of the world, special emphasis being laid on the history of mathematics in Poland.

The system of axioms is gradually developed during the study of deductive geometry.

Methods of Teaching Mathematics

Suggestions to teachers as to the ways in which mathematics may be taught, are contained in the mathematics syllabuses, but no special instructions exist recommending the use of any given method. Teachers dispose of a variety of methods for teaching each aspect of the subject, and are free to choose among them. The method most frequently used, especially for geometry, is the " genetic method of induction "; on the basis of experiment and examples, the pupils themselves discover properties and laws, which they then formulate, and prove the rough reasoning. This method is an activity method.

There are no special instructions relating to the methods to be used in the period of transition from primary to secondary education, for the simple reason that eleven-year comprehensive schools exist throughout the country. The change-over from intuition to abstraction is made gradually ; indications as to the degree of abstraction to be attained in each class are given in the syllabuses.

The teaching of mathematics is coordinated with that of the natural sciences, physics in particular. A knowledge of trigonometry is essential, for example, for the study in physics of harmonics and alternating current.

The materials used for mathematics lessons include drawing instruments, scissors, instruments for measuring and surveying (induction tables, slide rules, proportional dividers, pantographs, etc.), and models (cubes of a sum, loci, quadrilaterals with unequal sides, models illustrating theorems, solid polyhedrons, diagrams for rotatory machines illustrating the way solids of revolution are produced, etc.).

The practical work takes the form of problems and exercises in algebra and geometry. In conformity with activity methods, pupils make models in the mathematics workshop, relating to their work in class, under the supervision of pupils from higher classes.

A list of the textbooks approved for the coming year is published at the close of each school year in the Ministry of Education journal. Instructions concerning the use of mathematics textbooks, moreover, were published in September, 1952, by the centre for the further training of teachers.

Teachers

Secondary teachers of mathematics are required : (a) to have studied for five years in a university or a higher State school of education, and to have passed the corresponding diploma examination ; (b) to hold a first degree in mathematics.

There is a shortage of secondary teachers of mathematics, mainly on account of (a) the tremendous expansion of the country's economy and the resulting demand for specialists with higher qualifications, and (b) the increase in the birth rate and the consequent need for more general and vocational secondary schools.

In order to overcome this shortage, a number of higher schools of education have been established at the universities, admitting holders of the secondary school leaving certificate, and special and correspondence further training courses for primary teachers.

Further training for secondary teachers of mathematics is provided at district and *voivod* level by the centre for the further training of teachers.

All secondary teachers of mathematics must conform to certain instructions, and belong to an association which holds monthly meetings for the discussion of teaching methods. They are also able to refresh their knowledge through a pedagogical and scientific journal called " Mathematics ".

Throughout the year weekly and optional courses are organized in the large towns for the study of teaching and scientific problems.

Vacation courses are organized each summer in a given town, and are attended by four hundred secondary teachers of mathematics.

Miscellaneous

Work has now been in progress for two years on new mathematical syllabuses. The changes proposed at secondary level, affecting only the distribution of the subject matter, are as follows : (a) closer coordination of equations, functions, and inequalities ; (b) closer integration of algebra and physics ; (c) solid geometry to be taken in only one class. In order to adapt teaching to the evolution of mathematics, it is proposed to establish a more modern idea of the function and the functional value of the equation, so far as algebra is concerned, and to adopt elementary transformations (symmetry, revolution, and parallel translation) as a basis for teaching, in regard to geometry.

Research on the results obtained by the teaching of mathematics is regularly conducted by the centre for the further training of teachers, and on the syllabuses by the institute of education.

PORTUGAL

From the report sent by the Ministry of National Education

Place of Mathematics in Curriculum

Mathematics is taught in Portugal throughout the two-year first cycle and the three-year second cycle of secondary education, and in four sections of the two-year third cycle *(curso complementar)* preparing for higher education. The age range of the pupils is from 10 to 17.

Three hours a week (15% of the time-table) is allocated to mathematics in the first cycle (1st and 2nd years), three hours a week (12.5%) in the second cycle (3rd, 4th and 5th years), four hours a week (19%, 20% or 23.5%, according to section) in the third cycle (6th and 7th years, sections c, f, g and h).

Mathematics is designated as such in the curriculum.

Wherever it forms part of the curriculum, the subject is a compulsory one. It is included in promotion and leaving examinations on the same footing as other subjects.

Aims of Mathematics Teaching

The aim of teaching mathematics in the first secondary cycle is the practical and experimental one of developing a capacity to observe facts, reach general conclusions, and coordinate and classify experimentally determined properties, of arousing curiosity and interest, and of inculcating industry, studiousness, orderliness, and precision in the use of language. The aim of the subject in the second cycle is to develop through geometry a capacity for logical and deductive reasoning and clarity of ideas, and, in the third cycle, to teach the pupils to reason with speed and clarity.

The aim of teaching mathematics is thus not only to impart knowledge, but also to develop the initiative, reasoning power, and precision of thought and expression necessary for study at higher level.

Syllabuses

1st year. Arithmetic and geometry : the four operations and their main properties ; measurement of length, area and volume ; square root, angles, triangles, complex numbers, graphs. — *2nd year.* Arithmetic : the highest common factor and the lowest common multiple ; fractions, proportions, rule of three, percentages, simple interest. Geometry : triangles (relative magnitude of their elements, equalities) ; perpendiculars and obliques ; quadrilaterals, circumference, equivalent figures ; areas of plane figures and of polyhedrons, volume of polyhedrons. — *3rd year.* Algebra :

algebraic expressions, elementary concept of a function, operations with simple expressions and polynomials, algebraic fractions, simplifications, numerical equations of the first degree with one or two unknowns, graphs and problems. Plane geometry : angles, triangles (properties, equalities) ; angles of polygons ; constructions. — *4th year.* Algebra : algebraic fractions, numerical and literal equations of the first degree with one, two or three unknowns ; problems ; concept of infinitely small and infinitely large ; limit of a function. Plane geometry : circle, chord and perpendicular from the centre to a chord ; measurement of arcs and angles ; loci ; proportion ; similarity, similar polygons, regular polygons ; perimeter of the circumference ; area of plane figures. — *5th year.* Algebra : generalized concept of power ; irrational numbers, radicals, second-degree equations and problems ; arithmetical and geometrical progression. Solid geometry : relative positions of straight lines and planes, dihedral angle, distances, trihedral angle, regular polyhedrons ; lateral areas of prisms, pyramids, cylinders and cones ; surfaces of revolution, area and volume of polyhedrons. — *6th year.* Algebra : functions (geometrical representation) ; infinity and limits, derivatives of algebraical functions, algebraic fractions, symbols signifying impossibility and indeterminateness. Plane trigonometry : circular functions (definitions, variation, and graphical representation), fundamental trigonometrical equations, inverse circular functions. Rational arithmetic : theory of integral numbers and fundamental operations ; raising a quantity to a power ; systems of numeration ; prime numbers ; highest common factor and lowest common multiple. — *7th year.* Algebra : resolution of first-degree equations with one or two unknowns, biquadratic equations, and second-degree equations with graphs and discussion ; inequalities, problems, logarithms. Trigonometry : formulas for the sum and difference of two angles, and for logarithmic transformation ; finding the derivatives of circular functions from first principles, trigonometrical equations, resolution of right-angled and oblique-angled triangles ; calculation of areas, application to topography. Plane analytical geometry : cartesian and polar coordinates, distance between two points and the coordinates of the middle point of a segment ; loci, determination of the equations of some loci ; equation of the straight line ; equation and principal properties of the circumference, of an ellipse, a hyperbola and a parabola.

The textbooks used include bibliographical notes on famous mathematicians. Wherever necessary the teachers give information in a condensed form about the history of mathematics. The axiomatic aspects of the subject are dealt with in a broad way during the study of rational arithmetic and plane geometry in the second cycle.

Methods of Teaching Mathematics

According to official suggestions to teachers, the teaching of mathematics should be closely coordinated with that of art and handicrafts, in the first cycle (1st and 2nd year). In the third cycle many teachers apply the heuristic method.

The mathematics syllabuses for the 1st and, to a lesser extent, the 2nd year, were drawn up in such a way as to ensure a gradual transition

from primary to secondary education, and the methods in use at this level are also adapted to the same end.

The coordination of mathematics with other subjects is achieved mainly through close collaboration among the teachers. At the monthly staff meetings the teachers of a given year make decisions in common about the work to be done and ways of improving it.

The materials used for teaching mathematics include collections of weights and measures, three-dimensional geometrical figures, models for the study of solid geometry, and balances. Most secondary schools also possess projectors for films and filmstrips, etc.

Practical applications are given an important place in the mathematics syllabuses. The teaching of the subject, especially in the 1st and the 2nd year, as already noted, is closely integrated with that of art and handicrafts. In the second cycle the teachers choose problems relating to daily life. In the third cycle the teaching is mainly of a theoretical kind, but one hour a week is nevertheless allotted to practical exercises.

Official textbooks are used, but books of problems are chosen by the teachers themselves.

Teachers

Secondary teachers of mathematics are required to hold a first degree in mathematics. They receive professional training through the educational courses held in the arts faculties, covering general and educational psychology, pedagogy, history of education, and school hygiene. They obtain practical training through a two-year practical course in a training college. To be admitted to this college they must pass a fairly advanced entrance examination, and during the course they are required to attend a minimum of nine classes a week, take an active part in all teaching and administrative activities, etc. Those obtaining a minimum of 10 points on completing the course are entitled to sit for the State examination, with which their training ends.

Great difficulty is experienced in recruiting mathematics teachers, on account of the strong appeal made by other professions, such as engineering, and by an army, air force or naval career, and also on account of the severity of the entrance examination to the training college. In many cases teaching posts have to be filled with assistant teachers.

For a number of years the experiment has been made of exchanging secondary teachers from the metropolis with those of the overseas territories. Many secondary teachers obtain scholarships for further training in Portugal or abroad. Lectures, articles on mathematics in the various journals, etc., are also of value in this connection.

Miscellaneous

No changes are at present envisaged in the teaching of mathematics at secondary level.

Experimental work on the teaching of mathematics and written examinations is being carried out by the secondary education inspectorate and the training college.

ROUMANIA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

In accordance with the comprehensive curriculum adopted in 1948, mathematics is taught throughout the six years (classes V to X) of secondary education. The age range of the pupils is from 11 to 17 years.

The time-allocation for mathematics shows the importance attached to the subject. Six hours a week (20% of the total time-table) is allocated to it in the 1st year (class V), six hours (18.8%) in the 2nd year, five hours (15.7%) in the 3rd year, and six or seven hours (17.25%) in the 4th, the 5th and the 6th year. The average time allocated to mathematics throughout the course constitutes 17.7% of the total hours of instruction.

Mathematics is designated arithmetic, algebra, geometry, and trigonometry.

Mathematics is a required subject for all pupils. Pupils with special aptitude for mathematics can join the optional " mathematical circles ", in which they have opportunity to discuss and solve problems not contained in the syllabuses. Such pupils can also write articles for the " Mathematics and Physics Gazette ", a monthly published for secondary schools by the mathematics and physics society, and take part in the mathematics competitions organized at national level by this society.

Mathematics ranks as a major subject in the curriculum, and the pupils are examined in it at the end of each school year and on conclusion of their studies.

Aims of Mathematics Teaching

The general aims of mathematics teaching are given in the syllabuses now in force as being to develop the pupils' intellectual faculties, to give them basic mathematical knowledge, to foster and consolidate a habit of thinking logically, to develop the pupils' capacity to reflect, concentrate and generalize, to give them exercise in accuracy, and to cultivate their awareness of order and beauty.

The subject is also expected to impart the knowledge necessary for the study of subjects such as physics, astronomy, geography, and drawing, and to prepare the pupils to meet the problems of everyday life and for study at higher level.

Arithmetic is specifically expected to teach the pupils to apply the knowledge acquired to practical problems. Algebra is expected to widen the pupils' conception of number, and of functional dependence and its graphical representation, and to enable them to solve problems with the use of equations. Geometry is expected to develop the pupils' knowledge of space, and to teach them to apply geometrical properties to calculation, and to think logically.

Syllabuses

1st year (class V). Arithmetic : the operations with whole numbers, the properties of whole numbers, divisibility, the operations with vulgar and decimal fractions, the metric system, areas and volumes. — *2nd year (class VI).* Arithmetic : proportions, directly and inversely proportional quantities. Geometry : introductory work (point, line, area, angles, etc.), triangles, properties of triangles, congruence of triangles, dimensional relationships of the elements of the triangle, parallel lines and the corresponding theorems, angles of a polygon. Algebra : algebraical expressions, relative numbers, the solution of easy equations, graphs, the operations with monomials and polynomials, the decomposition of polynomials into terms. — *3rd year (class VII).* Algebra : algebraical fractions and the corresponding operations, proportion and proportional dependence, equations and inequalities of the first degree with two or three unknowns, the solution of problems with the use of equations, the graphical representation of the functions. Geometry : the parallelogram and the trapezium, the circle and exercises on construction, inscribed and circumscribed triangles and quadrilaterals. — *4th year (class VIII).* Algebra : powers and roots, the operations with irrational numbers, second-degree equations, second-degree trinomials, ratios between roots and coefficients, decomposition into linear terms, irrational equations, the functions and their graphical representation, systems of equations of the second degree with two unknowns. Geometry : similar figures, similarity of triangles and polygons, problems on graphical constructions, dimensional relationships, the theorem of Pythagoras, proportional lines in the circle, the algebraical method of solving geometrical problems, area of polygons, the ratio of the areas of similar triangles and polygons. — *5th year (class IX).* Algebra : arithmetic and geometrical progressions, limits, generalizing the concept of the exponent, the exponential and logarithmic function (graphs), logarithms, exponential and logarithmic equations. Plane geometry : inscribed and circumscribed regular polygons, the ratio of the perimeters of regular polygons, the determination of π, the ratio of the areas of two circles, the area of the sector and the circular segment. Solid geometry : lines and planes, the relationships between parallel and perpendicular straight lines and planes, the dihedral angle, polyhedrons. Trigonometry : the definition and value of trigonometrical functions, the relationships among trigonometrical functions, graphs of trigonometrical functions, goniometry, the logarithms of trigonometrical functions, the solution of the right-angled triangle. — *6th year (class X).* Algebra : complex numbers and the corresponding operations, the geometrical representation of complex numbers, inequalities, the solution of indeterminate equations and systems of indeterminate equations of the first and the second degree with one unknown ; equations of the first degree with one unknown and of the second degree with two unknowns ; the division of a whole polynomial in x by $(x \pm a)$; the solution of binomial equations of the third, the fourth, and the sixth degree ; the solution of trinomial equations. Geometry : polyhedrons, areas and volumes, solids of revolution, the sphere (the area of the spherical shell and the spherical zone), the solution of problems on solids of revolution, combinations of solids and of sections in the solids of revolution. Trigonometry : dimen-

sional relationships among the elements of any triangle (the theorem of sine, cosine, and tangent); the solution of the triangle (inverse trigonometrical functions, trigonometrical equations).

The mathematics teachers refer during the course to some aspects of the history of mathematics, and, briefly, to the life and work of the more famous mathematicians. They take special care to make their pupils understand the principles of axioms, including Euclid's axiom of parallel lines and also that of Lobatchevsky.

Methods of Teaching Mathematics

The Ministry of Education has issued a number of instructions concerning the methods to be used in the teaching of mathematics, with a view to the steady improvement of that teaching in all years of the course. A book called "Methodological instructions for mathematics homework" stresses the importance of individual work, of Pavlov's insistance on higher mental activity as a factor in the assimilation of mathematical knowledge, of the preparation of lessons, of evaluation of pupils' work, etc.

No special methods are used in the period of transition from primary to secondary education.

Correlation between mathematics and subjects such as physics, astronomy, line drawing, chemistry, and geography, is ensured (a) by correlating the syllabuses of the subjects involved, and (b) the use in mathematics lessons of suitable materials from other subjects. The mathematics syllabuses take due account of what the pupils need to know for the study of physics, astronomy, geography, etc.

A variety of teaching materials is used for mathematics lessons (charts, diagrams, apparatus, etc.). Teachers and pupils alike make sketches and drawings, construct geometrical solids, etc.

Theory is linked with practice by the application of theory to solving practical problems and by doing various kinds of practical work. The pupils are asked, for example, to solve problems involving the calculation of the area of fields and the area and volume of local constructions, in the 1st year (class V), to make use of geometrical ideas in field work, etc., in the 2nd and the 3rd year (classes VI and VII), and to apply their mathematical knowledge to simple topographical calculations, the calculation of distances and inaccessible heights, plan drawing, and the calculation of various areas and volumes, in the 4th, the 5th and the 6th year (classes VIII to X). The pupils themselves arrive at the conclusions, frame definitions, and verify certain laws.

Mathematics teachers throughout the country are required to make use of the textbooks and collections of exercises published by the Ministry of Education. The Ministry has issued instructions on the use of textbooks, covering the entire mathematics course, and giving special attention to certain difficulties met with in teaching the subject.

Teachers

Secondary teachers of mathematics are trained at higher level. They receive their academic and their professional training through the special

four-year course of a physics-mathematics faculty, or, in the case of teachers for the first three years (classes V to VII), through the two-year course at a teacher training institute. In the last year of their training they do teaching practice in a secondary school. They must pass the State examination for the university diploma of secondary teaching.

During the first five-year plan the teaching of mathematics in the first three years of secondary education (classes V to VII) was considerably developed. The number of places in the physics-mathematics faculties was increased, and it was then that the two-year teacher-training institutes were established. The course of these institutes has since been increased to four years.

Special institutes exist for the further training of secondary teachers. Every five years secondary mathematics teachers are required to follow a one-year refresher course, covering individual study and, when the schools are closed in the summer, courses and seminars lasting a month. The course concludes with an examination, on the basis of which the teacher is awarded a certificate, account of which is taken in the evaluation of his work. The individual study included in the course is based on a programme, valid throughout the country, and drawn up by the further training institute. The programme also contains a list of suitable books, and the topics it covers are dealt with in the courses and seminars held during the summer.

Each year this refresher course is attended by one fifth of the secondary mathematics teachers, and group and individual consultations, oral and written, on various aspects of the programme are organized by the further training institutes and regional offices of education. These bodies cooperate with the educational workers' syndicate, and organize mobile libraries of the books required for the course. During the four years preceding his participation in a refresher course, each teacher draws up a plan of individual study.

Secondary mathematics teachers also enjoy other regulation facilities for further training. In each school they form a group for the discussion of their professional problems, and to enable the elder ones amongst them to have an opportunity of giving the younger members of the staff the benefit of their experience. They also meet once a month in the pedagogical circles organized at urban, district, or communal level, for the discussion of problems of outstanding importance. Lessons are also given publically, with a view to improving the methods employed for the teaching of mathematics.

The teachers taking an active part in the work of further training are recruited from among the teachers doing outstanding work, and are required to follow special training courses lasting two or three months.

Conferences are held at district level twice a year, and all secondary mathematics teachers participate in them. Such conferences serve to make known the best methods, and give opportunity for a discussion of shortcomings and ways of remedying them, for placing selected teaching materials and aids on show, and for discussing the three-month work-plans and model lesson-plans. Other conferences are also organized by the Ministry of Education and the regional and district educational bodies, at which secondary mathematics teachers discuss scientific topics.

A number of specialist journals are available for mathematics teachers, such as "The Mathematics-Physics Gazette", "The Roumanian-Soviet Annals", and "Physics-Mathematics Series", and articles on mathematics are published in "The Pedagogical Journal", "The Teacher's Gazette", etc.

Miscellaneous

The Ministry of Education is contemplating the introduction of certain aspects of higher algebra into the mathematics syllabuses for the upper years. Arrangements have been made to control the written work of candidates for admission to the faculties and higher institutes of teacher training. The institute of pedagogical sciences is planning to introduce elements of polytechnical education into the teaching of mathematics, and experiments are being conducted in this connection in some of the schools.

SALVADOR

From the reply sent by the Ministry of Culture

Place of Mathematics in Curriculum

Secondary education lasts five years (pupils of 13 to 17 years of age), and is divided into a three-year lower cycle of general education *(plan básico)*, and a two-year upper cycle of preparation for the baccalaureate examination.

Mathematics is taught as a compulsory subject throughout the five-year course, for five hours a week in the lower cycle, and six hours a week in the upper cycle. The subject is designated *matemáticas (aritmética y geometría)* in the 1st *year*, *matemáticas (álgebra y geometría)* in the 2nd year, *geometría intuitiva*, *álgebra y trigonometría* in the 3rd year, *geometría plana y trigonometría* in the 4th year, and *geometría del espacio y cosmografía* in the 5th year.

The importance attached to mathematics is the same as that of other subjects, in promotion examinations, and depends on the type of baccalaureate (arts or science), in the leaving examination.

Aims of Mathematics Teaching

The aims of teaching mathematics in the lower cycle are (a) to enable the pupils to understand mathematical relationships through the use of symbols and operational techniques, (b) to reveal and guide their individual aptitudes, (c) to develop their capacity for observation, research, and criticism, (d) to cultivate their power of induction, (e) to enable them to express themselves concisely and exactly, (f) to enable them to apply mathematical reasoning to human activities, and (g) to enable them to understand and solve everyday problems.

The aims assigned to teaching mathematics in the upper cycle are to develop the pupils' power to think abstractly and logically, and for themselves, and to enable them to apply mathematical knowledge in the various fields of everyday life.

Syllabuses

1st year. Fundamental operations, decimal system, division, fractions, complex numbers, rule of three. Geometry : construction and measurement of straight lines, angles, arcs and polygons. — *2nd year.* Algebraical expressions, easy equations and formulas of direct interest, graphical representation, interpretation of and operations on polynomials, algebraical fractions, systems of equations, graphs ; triangles, problems and practical applications. — *3rd year.* Powers and roots of algebraical numbers, second-degree equations, practical applications. Intuitive geometry : straight

lines, angles, parallels and perpendiculars, loci, circumference, similarity and congruence, geometrical solids, measurement of areas. Trigonometry : trigonometrical functions of angles, relations in the elements of the circle, value of natural trigonometrical functions, solution of right-angled triangles, logarithms. — *4th year.* Definitions, postulate of the straight line and the plane, angles, triangles, etc., central and axial symmetry, dimensional relations, polygons, calculation of the area of plane figures, theorems and exercises on them. Trigonometry : general formulas and trigonometrical equations, graphical representation of functions, logarithms (properties), exponential equations, general properties of triangles, areas, problems and practical exercises. — *5th year.* Straight line and planes in space, relative position of straight lines and planes in space, dihedral angles, polyhedrons, the cylinder and right circular cone, the sphere, calculation of areas and volumes, problems and practical exercises. Cosmography : the celestial sphere, horizontal and equatorial coordinates, movement of the planets, dimensions, the earth, the sun, the moon, eclipses, the solar system, comets, the stars, the Milky Way, nebulae, parallaxes.

The syllabuses make no explicit reference to the history of mathematics, but teachers are recommended not to neglect this aspect of the subject in the course of their teaching.

Methods of Teaching Mathematics

Official suggestions on methods of teaching mathematics exist, which recommend teachers to use the projects method, to adapt their work to the general and individual level of intelligence of their pupils, to grade the difficulties of the subject, to reach abstract ideas through the prior use of concrete examples, to proceed inductively in the lower years, to apply the questions dealt with to everyday situations in social life in which the pupils are interested, to make their lessons primarily educational in nature, and to bear in mind that an understanding of mathematical questions is more important than automatic and mechanical mathematical skills. These suggestions also indicate how continuity may be ensured between the teaching at primary and at secondary level.

Teachers are recommended to make what materials they may need for their lessons. Films are available for the teaching of cosmography.

Frequent reference is made in the syllabuses to the practical applications of mathematics to the various fields of everyday life and human activities. Teachers are strongly recommended to give their pupils practical exercises before going on to formal work, but this recommendation meets with a number of obstacles in actual classroom practice.

Teachers are free to choose the textbooks they prefer, but are given advice in this connection by their headmasters, inspectors, etc.

Teachers

The majority of the mathematics teachers now serving have not had any special training. The minimum requirements for appointment are possession of a certificate from the teacher training school and passing

an examination in mathematics, or possession of the baccalaureate and passing an examination in mathematics and pedagogy. Some of the mathematics teachers hold engineering diplomas.

There is a shortage of mathematics teachers. For this reason the higher school of teacher training was set up in 1952; it will turn out its first batch of sixteen mathematics teachers at the end of 1955. The training course lasts five semesters, and comprises a total of 3,500 hours of lessons on academic and professional subjects. Students receive a scholarship which includes, in addition to free tuition, board and lodging, an allowance equal to a primary teacher's salary.

Secondary teachers may obtain further training through weekly meetings, correspondence courses, and short-term seminars, and information on teaching questions is made available to them. Every endeavour is also made by the secondary education directorate to guide and advise teachers, through its inspectors.

Miscellaneous

The reform of secondary education now being considered, will affect the mathematics syllabuses. These have been influenced by modern developments in mathematics, especially in regard to the theory of functions and the graphical representation of equations.

Two experimental schools are preparing to begin research work in the near future on the teaching of mathematics; in general it may be said that emphasis is laid on efficiency as a factor in the improvement of teaching methods.

SPAIN

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

Mathematics is taught as a compulsory subject throughout the four-year course (pupils of 10 to 14 years of age) at lower secondary level, for three hours a week. The subject then becomes compulsory in the 5th and the 6th year, only for the science section ; the time allotted to it is six hours a week in the 5th year, and three hours a week in the 6th. Instruction in mathematics is also given in the pre-university course. The time allocated to mathematics represents about 15% of the total time-table at lower secondary level, and 15%-25% at upper secondary level.

Mathematics appears in the curriculum as follows : *aritmética, geometría* (1st and 2nd years) ; *aritmética y álgebra, geometría* (3rd year) ; *álgebra, geometría* (4th year) ; *análisis, complementos de geometría, trigonometría* (5th year) ; *análisis* (6th year).

Generally speaking no special importance is attached to mathematics as compared with other subjects. In the science section, however, the subject occupies a leading place comparable to that of Latin in the literary section.

Aims of Mathematics Teaching

The aim of teaching mathematics is to develop the pupils' intelligence by enabling them to reason logically. The subject is also expected to equip them with the essential minimum of mathematical knowledge, which is then completed at upper secondary level and in the pre-university course by an introduction to mathematical analysis and by work preparing for such specialized higher studies as the pupils may have chosen.

Syllabuses

The official mathematics syllabuses are drawn up by the Ministry of National Education. The main points covered are as follows :

1st year. Exercises giving practice in the use of the decimal system of measurements ; the system of measurement of time ; simple problems in arithmetic and geometry. — *2nd year.* Positive numbers, whole numbers, fractions ; raising to a power ; extraction of square root ; character of divisibility ; variation ; plane geometry (1st part). — *3rd year.* Basic principles of the theory of numbers ; negative numbers ; algebraic expressions (polynomials not exceeding three terms) ; graphical representation of simple functions. Geometry : similar triangles ; theorem of Pythagoras and its applications ; inscribed and circumscribed regular polygons ; elementary

notions of trigonometry. — *4th year.* Fractional and negative indices ; calculations of roots ; second-degree equations ; second-degree trinomials ; graphical representation of functions (including exponential functions). Geometry : introduction to geometry in space, preparatory to a study of pure geometry ; diagrams ; application of knowledge to elementary problems of physics. — *5th year.* Mathematical analysis (permutation and combination, Newton's binomial theory) ; probability ; logarithms. Geometry : plane trigonometry. — *6th year.* Complex numbers ; introduction to infinitesimal calculus ; second-degree equations with complex roots ; study of the number " e ", elementary statistics (distribution and dispersion of the variable).

The secondary mathematics syllabuses make no special reference to the history of mathematics or to the principles of axioms. Model secondary schools exist, where provision is made for seminars and lectures on the history of mathematics for pupils following the pre-university course ; systematic use is made in this connection of wall charts showing the various epochs.

Methods of Teaching Mathematics

In addition to the secondary mathematics syllabuses, the Ministry of National Education has formulated the following suggestions with regard to the methods to be used for teaching the subject in each year. *1st year :* abstract reasoning should be avoided. — *2nd year :* the concepts of power, index, etc. should be introduced in an intuitive way, and with the help of numerous examples ; problems involving an application of the rule of three, etc., should in all cases correspond to reality ; the study of plane geometry should introduce the pupils to logical reasoning. — *3rd year :* the theory of numbers should be taught in abridged but logical fashion, and without burdening the pupil's mind with unnecessary theorems. — *4th year :* geometry should be so taught as to accustom the pupils " to see in space ", and with the help of numerous models of solid bodies ; objective reasoning should prepare the way to the later study of pure geometry ; the knowledge acquired should be applied to simple problems of physics. — *5th and 6th year :* while remaining in touch with reality, mathematics at the upper secondary level should become considerably more advanced, and be based on deduction ; teachers should reduce to a minimum the information they impart directly, and lead the pupils to think and discover as much as possible for themselves.

Coordination of methods to ease the transition from primary to secondary education, is undertaken only in schools with preparatory classes. No official recommendations have been made in this connection.

Textbooks approved by the Ministry of National Education are used for mathematics teaching. Each school also has a stock of teaching materials and auxiliary aids, such as geometrical solids, measuring instruments, apparatus demonstrating the rotatory notion, slide rules, calculating machines, and topographical materials. In addition, some model schools have special materials for the study of statistics and of mathematical curiosities, synoptic tables for the study of the various epochs of the history of mathematics, and projectors.

What is learned in mathematics lessons is continually applied in the fields of physics, and of economics, topography, and problems of daily life. The more formal aspects of the lessons, moreover, are supported by practical work, such as construction of various kinds, the use of apparatus, and surveying. Some of the model schools have an observatory for their pupils.

Textbooks must secure the prior approval of the education council *(Consejo de Educación)*.

Teachers

Secondary teachers of mathematics hold either a first degree or a doctorate in science. Candidates for the post of ordinary teacher are required to hold a first degree, are competitively selected, and must first have served as assistant teacher *(profesor adjunto)* in a secondary school ; candidates for the latter type of post are required to pass certain selective tests. No difficulty is experienced in recruiting mathematics teachers.

Mathematics teachers have various opportunities of securing further training, in the form of vacation courses (at Santander, for example), and conferences organized by the mathematical society *(Real Sociedad de Matemática)*, the higher council of scientific research *(Consejo superior de Investigaciones científicas)*, and the Jorge Juan Institute, and through the reading of specialist journals. There are also some scholarships for further training abroad.

Miscellaneous

No changes in the teaching of mathematics are at present envisaged. The modern development of mathematics has influenced the teaching of the subject only to a limited extent. In the model schools referred to above, experiments are made in teaching the subject, with groups of pupils of different intellectual levels.

Articles on methods of teaching mathematics are published by the specialist journals.

SWEDEN

From the reply sent by the Royal Board of Education

Place of Mathematics in Curriculum

Mathematics is taught (a) throughout the course of the four-year and the five-year lower general secondary schools *(realskola)*, (b) in the upper general secondary schools *(gymnasium)*, in the science section *(reallinje)*, the " social science option " sub-section and the first year of the " modern languages option " sub-section of the general modern section *(allmänlinje)*, and the 1st year of the Latin section *(Latinlinje)* and (c) throughout the course of the six-year and the seven-year general secondary schools for girls, except the last two years of the practical sections.

The age range of the pupils is from 11 to 15 years and from 13 to 16 years in the five-year and the four-year lower general secondary schools respectively, and from 15 to 19 in the upper general secondary schools.

The number of hours per week allotted to mathematics, and the percentage this figure represents of the total time-table (excluding hours for practical activities), are as follows:

Lower General Secondary Schools

Five-Year Course

Year	Hours	%
1st	4	16.6
2nd	5	20.0
3rd	3	11.5
4th	4	14.2
5th	4	13.8

Four-Year Course

Year	Hours	%
1st	4	15.4
2nd	3	11.5
3rd	4	14.2
4th	4	13.8

General Secondary Schools for Girls

Year	Hours	%
1st	4	18.2
2nd	5	21.7
2nd [1]	4	16.6
3rd	3	12.0
4th	3	12.5
5th	3	12.0
6th	3	—
7th	3	—

[1] The 1st year of the six-year course for girls coming from the 6th year of the primary school.

Upper General Secondary Schools

Science Section

(a) Four-Year Course

Year	Hours	%
1st	5	16.6
2nd	4	13.3
3rd : Biology option. .	4	13.3
Maths. option . .	7	23.3
4th : Biology option. .	4	14.3
Maths. option . .	7	24.1

(b) Three-Year Course

Year	Hours	%
1st	6	19.7
2nd : Biology option. .	4	13.3
Maths. option . .	7	23.3
3rd : Biology option. .	4	14.4
Maths. option . .	7	24.1

General Section

(a) Four-Year Course

Year	Hours	%
1st	5	16.6
2nd	4	13.3
3rd : Soc. studies option	4.5	15.8
4th : Soc. studies option	3.5	13.0

(b) Three-Year Course

Year	Hours	%
1st	6	20.0
2nd : Soc. studies option	4.5	15.8
3rd : Soc. studies option	3.5	13.0

Latin Section

Year	Hours	%
1st	3	10.0

Mathematics is designated as such, and is compulsory, in all years in which it is taught.

The subject has the same importance as other subjects in promotion examinations. There are oral and written papers in mathematics in the lower certificate examination *(realexamen)* and the higher certificate examination *(studentexamen)*; the written paper in mathematics is an option in the former examination, and compulsory only for pupils in the science section, and the social studies sub-section of the general section, in the latter examination.

Aims of Mathematics Teaching

The aims of mathematics teaching are given as follows in the curricula of State general secondary schools.

In lower general secondary schools, in accordance with the regulations of 1950, the subject is expected to extend the knowledge the pupils have

acquired at primary level, to enable them to apply such knowledge to various problems, those of a practical nature in particular, and to acquaint them with the elementary notions and methods of geometry.

In upper general secondary schools, in accordance with the regulations of 1954, mathematics is expected to extend and deepen the pupils' mathematical knowledge, and familiarize them with mathematical thinking, in the general section ; to impart knowledge corresponding in essentials with that imparted in the final year of the lower general secondary schools, in the Latin section ; and to extend and deepen the pupils' mathematical knowledge, and (especially in the mathematics sub-section) familiarize them with mathematical thinking, in the science section.

In the general secondary schools for girls, the aims of mathematics teaching are the same as for the lower general secondary schools.

Syllabuses

Lower General Secondary Schools

(a) *Five-Year Course. — 1st year.* Exercises on the four operations, with whole numbers, and practical applications (a continuation of the primary syllabus) ; formation, indication, multiplication and simplification of vulgar fractions, with examples ; the four operations with decimals, and practical applications ; exercises involving money, measures of length, areas, volumes, and weights ; mental arithmetic. — *2nd year.* The four operations with vulgar fractions, and practical applications ; percentages, calculation of revenue, profit and loss in buying and selling, discount, debentures and shares ; exercises involving the use of non-decimal units. Simple equations of the first degree, in figures, with one unknown, with practical applications ; mental calculations. Introduction to geometry. — *3rd year.* Equations of the first degree with one unknown, and practical applications (cont.) ; mental calculations. Geometry : points, straight lines, segments, angles, triangles, parallel lines, parallelograms (twenty theorems) ; simple problems. — *4th year.* First-degree equations (cont.), easy uses of the four operations with literal symbols ; mental calculations. Geometry : the circle, measures of segments, areas and angles, the perimeter and the area of a circle, arcs and sectors ; easy problems and calculations ; the drawing and interpretation of simple diagrams. — *5th year.* Literal equations of the first degree with one unknown, systems of simple equations of the first degree with two unknowns ; extraction of square roots ; the solution of practical problems with and without the use of equations ; mental calculations. Geometry : the theorem of transversals, similarity, scales of length and area, the theorem of Pythagoras, problems of plane and of solid geometry.

(b) *Four-Year Course. — 1st year.* A continuation of the primary course on fractions ; equations of the first degree with one unknown, and practical applications ; exercises on the use of units ; mental arithmetic. Introduction to geometry, practical work. — *2nd year.* As for the 3rd year of the five-year course. — *3rd year.* As for the 4th year of the five- year course. — *4th year.* As for the 5th year of the five-year course.

UPPER GENERAL SECONDARY SCHOOLS

(a) *General Section (four-year course). — 1st year.* The operations with literal symbols, involving negative numbers, first-degree equations, systems of equations of the first degree with one or more unknowns, square roots and irrational numbers, equations of the second degree with one unknown ; simple equations involving radicals ; the study, involving the use of rectangular coordinates, of functions of the first and the second degree ; a general revision of the lower general secondary geometry course ; principles of equations and similarities as applied to problems of plane geometry. — *2nd year.* Relationships between the roots and the coefficients of an equation of the second degree, division of the factors of a polynomial of the second degree, equations of a higher degree ; short explanation of roots, powers, and logarithms ; the slide rule. Plane geometry, introduction to solid geometry. — *3rd year (social studies option).* Arithmetical and geometrical progressions ; simple problems on compound interest ; easy notions of the calculation of probabilities ; some basic notions of statistics ; trigonometrical calculations relating to triangles ; a short treatment of the straight line by analytical geometry ; a revision and an extension of the principles of equations with a view to a study of the theory of functions ; notions of limits and derivatives ; the study of rational functions and the construction of the corresponding curves. — *4th year (social studies option).* Conclusion of the theory of functions ; short course in solid geometry, the calculation of areas and volumes with the aid of derivatives and integrals ; the binomial theorem.

General Section (three-year course). — 1st year. The combined syllabuses of the 1st and the 2nd year of the four-year course. — *2nd year (social studies option).* As for the 3rd year of the four-year course. — *3rd year (social studies option).* As for the 4th year of the four-year course.

(b) *Latin Section. — 1st year.* Equations of the first degree with one or two unknowns, and simple practical applications ; notions of square roots ; theorems and constructions relating to transversals and similar triangles ; problems of plane and solid geometry.

(c) *Science Section (four-year course). — 1st and 2nd years.* As for the 1st and the 2nd year of the general section (four-year course). — *3rd and 4th years (biology option).* As for the 3rd and the 4th year of the general section (four-year course, social studies option). — *3rd year (mathematics option).* As for the 3rd year of the science section (biology option), plus the following : a short treatment of the circle by analytical geometry ; introduction to solid geometry. — *4th year (mathematics option).* Treatment of the parabola, the ellipse and the hyperbola by analytical geometry ; conclusion of the solid geometry course ; the study of the functions and their curves (cont.) ; the indeterminate integrals and their applications to the calculation of areas and volumes ; mathematical induction ; the binomial theorem ; general revision of the work done in previous years.

Science Section (three-year course). — 1st year. The combined syllabuses of the 1st and the 2nd year of the four-year course. — *2nd and 3rd years (biology option).* As for the 3rd and the 4th year of the four-year course (biology option). — *2nd and 3rd years (mathematics option).* As for the 3rd and the 4th year of the four-year course (mathematics option).

General Secondary Schools for Girls

(a) *Seven-Year Course. — 1st year.* Exercises on the four operations, with whole numbers, and practical applications (cont.), introduction to fractions, exercises and problems involving practical applications on the use of money, and units of length, area, volume, and weight ; mental arithmetic. — *2nd year.* Conclusion of the work on fractions ; percentages ; proportional parts of materials, profit and loss, discount, buying and selling, income, debentures and shares ; non-decimal units ; mental arithmetic ; practical work involving measurements, weighing, and the determination of densities ; introduction to geometry. — *3rd year.* Simple equations and their applications ; mental calculations. Geometry : the straight line, segments, angles, triangles, easy practical theorems, simple problems. — *4th year.* Equations and their applications, introduction to algebraical expressions ; mental calculations. Geometry : parallel lines, parallelograms, introduction to the study of the circle, simple theorems and problems. — *5th year.* Equations of the first degree with one unknown, and their applications ; simple uses of the four operations with literal symbols ; mental calculations. Geometry : conclusion of the work on the circle, practical theorems and easy problems ; drawing simple diagrams. — *6th year.* Systems of equations of the first degree with two unknowns, and their practical applications ; extraction of square roots ; the operations with literal symbols ; mental calculations. Geometry : the theorem of transversals, similarity, scales of length and area, the theorem of Pythagoras ; problems of plane and of solid geometry. — *7th year.* Simple functions ; the slide rule (multiplication and division) ; drawing diagrams, in particular those of frequencies ; practical problems, especially those involving shares, foreign currency, and compound interest ; mental calculations.

(b) *Six-Year Course. — 1st year.* Continuation of the work done at primary level on fractions and percentages ; exercises on the units ; mental arithmetic ; practical work involving measurements, weights, and the determination of densities. Introduction to geometry. —*2nd, 3rd, 4th, 5th and 6th years.* As for the 3rd-7th years of the seven-year course.

The history of mathematics is not included in the syllabuses, but teachers make reference to it in the course of their work, and pupils at the upper general secondary schools have opportunities of securing private tuition in it. The principles of axioms are also not systematically included in the mathematics course, but pupils in the upper general secondary schools are given an introduction to them.

Methods of Teaching Mathematics

Recommendations concerning methods of teaching mathematics are given in an article that appeared in *Aktuellt* (No. 4, 1951), the bulletin of the Board of Education, as regards the lower general secondary schools, and in an article that appeared in *Aktuellt* (No. 18, 1954), as regards the upper general secondary schools.

The first of these two articles indicates what teachers should do to facilitate the transition from primary to secondary education.

The teaching of mathematics is coordinated with that of other subjects. This is especially so with regard to physics, and the mathematical knowledge required for the study of given aspects of this subject is always taught suitably in advance. The same may be said of drawing. The problems of coordination are discussed at the conferences held on the various subjects in the secondary curriculum.

As regards the materials and aids used for the teaching of mathematics, it is of interest to note that stereometric models are generally used, and that the use of the slide rule is compulsory in the upper classes. There are some Swedish films for the teaching of mathematics, but little use is made of them.

The extent to which the mathematics course includes practical applications is indicated in the articles, referred to above, giving recommendations on the methods of teaching the subject. Details are there given of how teachers may prepare for the theoretical treatment of certain aspects of the subject through giving practical exercises.

As for all other subjects, teachers are required to make use only of the mathematical textbooks and tables given in an official list compiled by the State committee on school textbooks.

Teachers

Secondary teachers of mathematics are required to possess equivalent qualifications to those of the teachers of other subjects. The teaching of mathematics is generally combined with that of physics, chemistry or biology. An assistant teacher *(adjunkt)*, who works mainly at lower secondary level, is required to hold the *filosofie magister* degree, roughly the equivalent of the teachers' certificate, and generally three and in no case less than two certificates in mathematics, and to have had a minimum of two years' teaching experience. A teacher *(lektor)* in an upper general secondary school is required to hold the same academic qualifications as an assistant teacher, plus the *filosofie licentiat* degree and not less than two certificates in mathematics. A doctorate is not essential.

There is a shortage of secondary teachers, especially for mathematics and natural science, and in many cases it has been necessary to appoint primary teachers, engineers, etc. The most important step taken to overcome the shortage has been to give encouragement to primary teachers to undertake further training, and to establish a category of teacher between the primary and full secondary categories, and to place primary teachers with some university training in that category. Short courses have also been organized, especially for mathematics.

Mathematics teachers are rarely if at all included in the otherwise frequent exchanges of teachers with other northern countries. The government makes special grants for further training in the secondary subjects, including mathematics, covering travelling and maintenance expenses. The organization of more important further training facilities for mathematics teachers is envisaged. The association for the teaching of mathematics and natural science organizes lectures and short-term conferences. A group of scientists and secondary mathematics teachers publishes a journal called *Elementa*.

Miscellaneous

Through the reform of upper general secondary schools effected in 1953, the last two years of the science section of these schools were divided into biology and mathematics sub-sections, and changes were made in the mathematics syllabuses, including added emphasis on the theory of functions, in conformity with recent developments in the mathematical field, and the inclusion of a course on statistics.

The experimental work in secondary schools undertaken by a special committee of the Board of Education concerns the teaching of mathematics to some extent. The Board, moreover, recently began a study of the teaching of elementary mathematics, with a view to standardizing terminology and the presentation of problems. Detailed recommendations on the methodology of mathematics teaching were drawn up on the basis of experimental work.

SWITZERLAND

Geneva

From the reply sent by the Department of Education

Place of Mathematics in Curriculum

Mathematics is taught throughout (a) the lower secondary cycle (pupils 12-13 to 15 years of age), and (b) the upper secondary cycle (pupils 15 to 19 years of age).

The hours per week allotted to the subject, and the percentage these hours represent of the total time-table, are as follows:

Boys' secondary school. 3 hours (9.13%) in each of the three years of the lower cycle. 3 hours (10%), 4 hours (13%), 3 hours (10%), and 3 hours (9.13%) in the 1st, the 2nd, the 3rd, and the 4th year respectively of the classical, Latin, and modern sections in the upper cycle, and 5 hours (16.6%), 5 hours (16.6%), 8 hours (25%), and 9 hours (29%) in the 1st, the 2nd, the 3rd, and the 4th year respectively of the science section.

Girls' secondary school. 3 hours (10 to 11.5%) in the two years of the lower cycle. 3 hours (10.3%), 4 hours (13.3%), 3 hours (10%), and 3 hours (10%) in the 1st, the 2nd, the 3rd, and the 4th year respectively of the modern Latin section in the upper cycle; 4 hours (13.3% to 14%) in each of the four years of the modern section; 2 hours (10%), 3 hours (15%), 2 hours (10%) in the 1st, the 2nd, and the 3rd year respectively of the four years of the general section. No mathematics are taught in the last year of this section.

Modern secondary school. 5 hours (15.6%) in each of the two years.

Mathematics is a required subject in all classes in which it is taught. It is designated *mathématiques*, and, in the last two years of the science section of the boys' secondary school, *mathématiques et géométrie descriptive*.

Special importance is attached to mathematics, as compared with other subjects, in the schools or sections with a technical bias, such as the modern secondary school, the science section of the boys' secondary school, and the technical secondary school.

Aims of Mathematics Teaching

The aims of mathematics teaching are not fixed in any official text, but may be regarded as being the imparting of knowledge and skills, the development of intellectual aptitudes (the scientific spirit, precision, clarity, etc.) and moral capacities (honesty, conscientiousness, etc.).

Syllabuses

Boys' secondary school.

Lower cycle. — 1st year (class VII). Arithmetic : whole numbers, resolution into prime factors, fractions. Geometry : angles, loci, constructions, triangles, quadrilaterals, regular polygons, their construction. — *2nd year (class VI).* Arithmetic : vulgar fractions (revision of the four operations, problems), square root, the metric system, ratios, percentage, density. Geometry : the circumference, the theorem of Pythagoras, areas and volumes. — *3rd year (class V).* Arithmetic : ratios and proportions, the rule of three, proportional parts, alloys and mixtures, simple interest, discount. Geometry : congruent, equivalent, and similar figures ; the trunk of pyramids and cones ; areas and volumes. Algebra : simple uses of the four operations, numerical equations of the first degree, problems.

Upper cycle. Classical, Latin, and modern section. — 4th year (class IV). Arithmetic : revision of the operations and their properties. Algebra : algebraical calculation, numerical and literal equations of the first degree with one or more unknowns, inequalities of the first degree. — *5th year (class III).* Algebra : equations with one unknown, resolution of the trinomial into factors, the algebraical and graphical study of the function of the second degree, biquadratic and irrational equations, systems of the second degree. Logarithmic calculation. Plane geometry : triangles, polygons, the circle, symmetry, the homology and the similarity of figures, perimeters and areas. — *6th year (class II).* Algebra : progressions, compound interest, combinatory analysis, the binomial theorem of Newton. Solid geometry : the perpendicularity and parallelism of straight lines and planes ; dihedrons and polyhedrons ; prisms, pyramids, the cylinder, cones and spheres, areas and volumes. Trigonometry : trigonometrical relations in the right-angled triangle and circular functions, vectors, the addition of arcs, trigonometrical identities and equations, the solution of triangles. — *7th year (class I).* Plane analytical geometry. Infinitesimal calculus : derivates, the variation of a function, limits and inflexions, areas, volumes of revolution. *Science section. — 4th year (class IV).* Arithmetic and algebra : as for the 4th year of the classical section. Plane geometry : as for the 5th year of the classical section. — *5th year (class III).* Algebra and logarithmical calculation : as for the 5th year of the classical section. Additional algebra : progressions, artifices of calculation, the auxiliary variable. — *6th year (class II).* Algebra : compound interest, annuities, depreciation, combinatory analysis, Newton's binomial theorem. Infinitesimal calculus : the derivate of a function of a variable, the variation of a function, limits and inflexion, determinants of the second and the third order. Trigonometry : as for the 6th year of the classical section. Descriptive geometry : orthogonal projections ; problems involving the point, the straight line, the plane, trihedrons, formulas of spherical trigonometry, polyhedrons, shadows. — *7th year (class I).* Algebra : numerical and whole series ; derivates of inverse circular functions and exponential and logarithmic functions ; differential, integral calculus ; lengths, areas, and volumes ; curves. Analytical geometry : as for the 7th year of the classical section. Descriptive geometry : curved surfaces, the cone, the cylinder, the sphere, solids of revolution, transferred shadows, projective geometry.

Girls' Secondary School.

Lower cycle, Latin and modern sections: the syllabuses are similar to those of the lower cycle and the classical, Latin, and modern sections of the boys' secondary school, but are distributed over six years instead of seven. *General section.* The syllabuses are simplified. — *3rd year (class IV).* Algebra : elementary algebraical calculation, equations and problems of the first degree with one unknown. — *4th year (class III).* Algebra : equations of the first degree with two unknowns, inequalities of the first degree, the concept of function, graphical representation of functions of the first degree, powers and square root. Geometry : some of the main theorems of plane geometry ; the circle, the isosceles triangle, the congruence of triangles, the angles formed by two parallels and a secant, loci, quadrilaterals, polygons, areas. — *5th year (class II).* Algebra : equations of the second degree, arithmetical and geometrical progressions. Geometry : ratios and proportions, homology, geometrical constructions of common curves.

Modern Secondary School.

1st year. Arithmetic : fractions, revision of the metric system, square root, compound numbers (time), ratios (scales, density, shares, rates, simple interest, percentages). Geometry : loci, triangles, quadrilaterals, polygons, constructions, the circle, the area of plane figures, the volume of parallelopipeds and cylinders. — *2nd year.* Algebra : elements (operations, literal calculation, algebraical fractions), equations and problems of the first degree with one or two unknowns, equations and problems of the second degree with one unknown. Geometry : equivalence, area and volume of solids, the theorem of Pythagoras, similarity. Trigonometry : introduction, the solution of right-angled triangles.

The secondary mathematics syllabuses make no explicit reference to the history of mathematics or to the principles of axioms.

Methods of Teaching Mathematics

There are no official instructions or recommendations concerning methods of teaching mathematics. This is a matter for the discretion of the teacher.

There are no special instructions concerning the methods for use in the period of transition between primary and secondary education. The syllabuses pay due heed to correlation between the various levels of education (primary, lower secondary, and upper secondary) and to the correlation of mathematics with other subjects. The teachers of the various subjects, moreover, themselves take steps to ensure correlation ; they form committees for each subject and each level of education.

The teaching materials used for mathematics lessons consist of some geometrical solids.

Mathematics is applied practically through the choice of practical problems involving interest, foreign exchange, depreciation, surveying, etc. Little use is made of activity methods.

A list of textbooks available for each school is published annually.

Teachers

Secondary teachers of mathematics are required to have had academic training (they must hold at least a first degree in mathematics) and professional training (they must possess the teaching certificate complementing their degree, awarded on the basis of attendance at university courses in psychology and methodology, and of six months' practice teaching in a public secondary school).

No difficulty has so far been experienced in recruiting secondary teachers of mathematics.

The mathematics teachers themselves organize meetings and lectures for the purpose of further training, and they are affiliated to the Swiss secondary mathematics teachers' association. The Department of Education, moreover, grants facilities (allowances, paid leave) to teachers wishing to attend meetings, conferences, etc., in Switzerland or abroad, and for the exposition of new methods.

Miscellaneous

The Department of Education is engaged on the revision of the lower secondary curriculum. A committee for each subject, including mathematics, composed of representatives from the various schools, has been appointed to carry out the revision, and to draw up an introduction for the guidance of teachers, covering aims, teaching methods, etc.

The rate at which modern developments in the mathematical field is allowed to affect the teaching of the subject at secondary level, is on the whole slow and prudent.

A number of inquiries on the secondary teaching of various subjects, mathematics in particular, have been carried out by students at the *Institut des sciences de l'éducation*.

Ticino

From the reply sent by the Department of Public Education

Place of Mathematics in Curriculum

Secondary education covers a period of eight years, and is given in four-year lower general secondary schools *(ginnasio inferiore)* (pupils aged 11 to 15 years) and four-year upper general secondary schools *(ginnasio o liceo)* (pupils aged 16 to 19 years). The present inquiry does not include the teacher training schools *(scuola magistrale)* existing at upper secondary level.

Five hours a week is allocated to mathematics in the 1st year, and four hours in the 2nd, the 3rd, and the 4th year, in the lower general secondary schools, and three hours a week (9-10% of the total time-table) in all four years of the arts section, and five hours (15.6%) in the 1st year,

six hours (18.7%) in the 2nd year, and seven hours (21.2%) in the 3rd and the 4th year of the science section, in the upper general secondary schools.

Mathematics is invariably a required subject. The subject is designated *matematica*, except in the 3rd and the 4th year of the science section, in which it is called, respectively, *algebra complementare e geometria analitica* and *algebra elementare e geometria descrittiva*.

After Italian, mathematics is the most important subject in both promotion and school leaving examinations.

Aims of Mathematics Teaching

The aims of mathematics teaching at secondary level are to contribute to the pupils' general education and, in the scientific section, to prepare the pupils for study at higher level.

Syllabuses

Upper general secondary schools, arts section. *1st year:* equations and systems of equations of the first degree, rational Euclidean geometry. — *2nd year:* real numbers, second-degree equations, congruent plane figures, the calculation of areas. — *3rd year:* problems of the second degree, solid geometry. — *4th year:* progressions, logarithms, exponential equations, trigonometry, Cartesian analytical geometry.

Upper general secondary schools, science section. *1st year:* as for the arts section. — *2nd year:* real numbers, complex numbers, equations and systems of equations of the second and higher degrees, analysis, ratios and proportions, similarity, the measurement of the circle. — *3rd year:* logarithms, determinants, rational functions and their derivates, equations of the third and the fourth degree, solid geometry, plane and solid analytical geometry, vectorial calculus, trigonometry. — *4th year:* infinitesimal calculus, differential geometry, descriptive geometry.

The history of mathematics is not included in the syllabuses, but is occasionally referred to by teachers in the course of their lessons.

Methods of Teaching Mathematics

There are no official instructions concerning methods of teaching mathematics, or the period of transition from primary to secondary education. The correlation between mathematics and other subjects is ensured by the teachers themselves.

Teachers

Secondary teachers of mathematics are required to have completed their university studies and to hold the secondary teacher's diploma.

So far there has been no difficulty in recruiting them.

No official provision is made for the further training of secondary teachers, except for the award of scholarships for that purpose. This is a matter which is left to the initiative of the teachers themselves.

SYRIA

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

The secondary course is a seven-year one, divided into a four-year lower or middle school cycle, and a three-year upper cycle of preparation for study at higher level. For admission to the lower and upper cycles, pupils are required to be not under 10 or over 15 years of age, and not under 14 or over 20 years of age, respectively.

Mathematics is taught as a compulsory subject throughout the secondary course. Out of a weekly total of twenty-eight hours, the time allocation for mathematics is four hours in the lower cycle, and, in the upper cycle, three hours in the arts section and six hours in the science section.

Mathematics is designated in the syllabuses as follows : arithmetic and elementary algebra in the 1st and the 2nd year ; algebra and plane geometry in the 3rd and the 4th year ; solid geometry and algebra in the 5th year ; elementary infinitesimal calculus, trigonometry, and theoretical arithmetic in the 6th year ; mechanics, descriptive astronomy, conic sections, and descriptive geometry in the 7th year.

Mathematics is regarded as one of the most important subjects in the secondary curriculum ; the work done in the subject must always conclude with an examination.

Aims of Mathematics Teaching

The aims of teaching mathematics may be be summed up as being to cultivate the pupils' capacity to think, to accustom them to exactitude, and to prepare them for the study of natural science at university level.

Syllabuses

1st year. Problems involving the four operations, fractions, measurement, construction of simple geometrical figures. — *2nd year.* Proportions, square roots, simple interest, first-degree equations. — *3rd year.* First-degree equations with one or two unknowns, triangles, quadrilaterals, the circle and corresponding theorems. — *4th year.* Second-degree equations with one or two unknowns, graphs and curves, similar figures, regular polygons, areas. — *5th year.* Straight lines, planes in space, prisms, cylinders, pyramids, cones, spheres, parametrical algebra, equations, inequalities, powers, logarithms. — *6th year.* Derivatives and curves, sine, cosine, tangent, cotangent, sine of the sum of two arcs, trigonometrical equations,

solution of triangles, functions of multiples and submultiple of angles, errors. — *7th year.* Vectors, speed, acceleration, motion, equilibrium, forces; the celestial sphere, the planets, the sun, the moon, the stars; parabola and hyperbola, inversion, homologous transformation, descriptive geometry.

The syllabuses do not include the history of mathematics.

Methods of Teaching Mathematics

Suggestions to teachers concerning methods of teaching mathematics in the various years are issued by the Ministry of Education. The inspectors also give teachers advice in this connection, and help with the solution of their individual problems.

Continuity of teaching as between primary and lower secondary (middle school) education is ensured through careful grading of the syllabuses.

The teaching of mathematics is closely integrated with the teaching of other science subjects, and their mutual dependence is kept in view when the respective syllabuses are being drawn up. Due account is taken, for example, of the fact that the pupils will need certain algebraical concepts for the study of given formulas for physics.

Some teaching aids and very simple demonstration apparatus are used for the teaching of mathematics. Very little practical work is done, owing to the lack of time.

Textbooks are published at national level by the Ministry of Education. The Ministry holds the authors' rights, and facilitates the purchase of textbooks by charging reduced prices. The textbooks thus published are selected from among the best available.

Teachers

Secondary teachers generally hold a university degree. The Syrian University trains qualified teachers of mathematics holding a diploma in their subject and a teacher's certificate.

Teachers are in short supply at all levels of education, as a result of a considerable increase of the school population and of the number of schools. To remedy the position, foreign teachers have been appointed in certain cases, teachers have been paid for extra work, and primary teachers have been used at lower secondary (middle school) level.

Teachers obtain further training through the award of scholarships for study abroad, and the issue of instructions, by the education authorities, and through the work of the inspectors.

THAILAND

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

Mathematics is taught as a required subject throughout the three-year lower secondary cycle, and the three-year upper secondary cycle preceding the pre-university course, under the designation of arithmetic, algebra, and geometry. Four hours a week (13.33% of the total time-table) are allocated to the subject in the lower cycle, and six hours a week (20% of the total time-table) in the upper cycle.

No special importance is attached to the teaching of mathematics in comparison with other subjects.

Aims of Mathematics Teaching

The aims of the teaching of mathematics at secondary level are to promote concentration, accuracy, carefulness, rapidity, orderly thinking, confidence, and experience in meeting and solving problems.

Syllabuses

Lower cycle. Arithmetic : Thai, metric and English measures of weight, capacity and length, currency exchange rates between Thailand and countries with which it has trade relationships, proportion, percentages, simple interest, areas of some polygons ; accounts, invoices, bills, and receipts. Geometry : straight lines, angles, parallel lines, polygons, circles and the circumference, constructions, ratio and proportion, problems concerning triangles. — *Upper cycle.* Arithmetic : fractions, ratio and proportion, partnership percentages, simple and compound interest, square and cube roots, the calculation of areas and volumes. Algebra : algebraical symbols, negative quantity and the operations involved, simple equations and problems, graphs, functions and the use of factors, lowest common multiple and highest common factor, elementary operations (up to complex fractions), axioms, definitions. Geometry : theorems and constructions of straight lines, angles, triangles, parallel lines, polygons, and circles ; areas of rectangles, triangles, and polygons.

The mathematics syllabuses include an introduction to axioms, but not the history of mathematics.

Methods of Teaching Mathematics

Suggestions have been officially issued concerning the methods to be used in the teaching of mathematics in secondary schools, and, occasionally,

on teaching methods suitable for the transfer stage from primary to secondary education. The methods in general use by teachers are those they have become familiar with during their teacher training.

The authorities endeavour to correlate the teaching of mathematics with that of experimental physics, a required subject, by including in the mathematics syllabuses the pre-requisite materials for the study of experimental physics.

Few materials or auxiliary aids are available for the teaching of mathematics, in comparison with other subjects; they do not generally include films, slides, etc.

The mathematics syllabuses do not yet include applied mathematics, but the inclusion of elementary statistics in the near future is envisaged, as this subject is in demand.

The general approach to the teaching of mathematics is still the traditional one, and the use of activity methods is still only in the experimental stage.

School textbooks are prescribed by the central educational authorities.

Teachers

The minimum requirement for appointment as a secondary teacher of mathematics is possession of the post-primary teacher's certificate, which demands three years of training after completion of the upper secondary cycle. The average standard of such teachers is possession of the secondary teacher's certificate, which calls for five years of training following completion of the upper secondary cycle. Few university graduates are available for teaching mathematics in secondary schools. No difficulty is experienced in recruiting secondary mathematics teachers.

Serving teachers keep up-to-date in their knowledge through an in-service training programme, university courses for external students, and the correspondence courses offered by the teachers' association.

Miscellaneous

The teaching of mathematics at secondary level appears to be the last stronghold of the traditional approach, and no modifications are at present envisaged in it.

UKRAINE

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

The ten-year schools are commonly called "secondary schools", but the secondary stage really begins only with the 5th year. Secondary education thus lasts six years, from the 5th to the 10th year inclusive, the age range of the pupils being from 11 to 17 years.

Mathematics is taught throughout the secondary stage. Six or seven hours a week (roughly 20% of the total time-table) are allocated to the subject. The subject embraces arithmetic in the 1st year (the 5th year of schooling), arithmetic, algebra and geometry in the 2nd year, algebra, geometry and technical drawing in the 3rd and the 4th year, and algebra, geometry, trigonometry and technical drawing in the 5th and the 6th year. The subject is a compulsory one. Pupils may take up an additional and optional study of mathematics in their school's mathematics society.

Mathematics ranks as a major subject. Pupils take a written and an oral examination in it at the end of the 1st, the 4th, and the 6th year.

Aims of Mathematics Teaching

The aims of mathematics teaching at secondary level are as follows: to give the pupils a groundwork in mathematics; to enable them to understand the quantitative aspects of the real world and its forms in space; to impart habits of orderliness and accuracy in reasoning and drawing conclusions; to develop the pupils' power to concentrate and their liking for individual work; to prepare for the study of mathematics at higher level; to enable the pupils to apply mathematical knowledge to practical problems, and to create conditions favouring the free choice of an occupation.

Syllabuses

1st year (5th year of schooling). The beginning of a methodical course in arithmetic (the laws of the operations, the divisibility of numbers, vulgar and decimal fractions). — *2nd year.* Arithmetic: interest, proportions. Algebra: algebraical expressions, positive and negative numbers, the first three operations on monomials and polynomials, introduction to equations of the first degree. Geometry: the beginning of a methodical course; general definitions, straight lines, curves, the circumference, angles, triangles, parallel straight lines. — *3rd year.* Algebra: the division of monomials and polynomials; fractions and algebraical proportions,

equations of the first degree in one, two and three unknowns, the corresponding graphs. Geometry : quadrilaterals, the circumference, inscribed and circumscribed triangles and quadrilaterals. Technical drawing : the beginning of a methodical course. — *4th year*. Algebra : degrees and roots, equations of the second degree, biquadratic equations, functions and graphs, equations of the second degree in two unknowns. Geometry : homology and similarity, trigonometrical functions of the acute angle, dimensional relationships of the elements of the triangle and the circle, the area of polygons. Technical drawing : projections and mensuration. — *5th year*. Algebra : progressions, degrees, functions, logarithms. Geometry : the measurement of planes, solid geometry (as far as polyhedrons). Trigonometry : the beginning of a methodical course ; definition of the trigonometrical functions of any angle, radian, formulas of conversion, trigonometrical formulas of the numerical argument, the use of logarithmic tables, the solution of right-angled triangles. Technical drawing : pyramids, cones, spheres, prisms, cylinders ; sections of the foregoing ; the ellipse and its projections. — *6th year*. Algebra : combinations, complex numbers, inequalities and their properties, equations in several unknowns. Geometry : polyhedrons, solids of revolution. Trigonometry : correlations ; theorems of sine, cosine and tangent ; functions and trigonometrical equations. Technical drawing : the determination of the form of sections by the rotation of the solids of revolution.

The history of mathematics is dealt with in the course of the work, but is not given separate treatment. Suggestions concerning the teaching of the history of mathematics are to be found in the specialist journals.

Methods of Teaching Mathematics

Official recommendations concerning the teaching of mathematics at secondary level are to be found in the mathematics syllabuses, in circulars issued by the Ukrainian Ministry of Education, and in the annual analysis of the results obtained by the pupils. They cover the aims and methods of mathematics teaching, an analysis of the mathematics syllabuses, and the use of textbooks, and state that no one method can be regarded as universally valid. The teacher has to choose the method best suited to the age of the pupils and the topic being dealt with.

Official instructions concerning the transition from primary to secondary education are to be found in the handbooks on the teaching of mathematics at secondary level, in the mathematics syllabuses, and in the circulars on methods issued by the Ukrainian Ministry of Education.

The mathematics teachers give attention to the question of correlating mathematics with other subjects, but their main concern is nevertheless to give a coherent and suitably graded presentation of the various aspects of mathematics.

The materials provided by the State for mathematics lessons include models of geometrical solids, apparatus and working models showing the relationship between the elements of each figure, various charts and diagrams, and instruments and apparatus for practical work (logarithmic tables, calculating machines, compasses, protractors, theodolites, clinometers, astrolabes, micrometers, etc.).

Films are also available, illustrating the trigonometrical functions, the solids of revolution, the mensuration of solids, etc. Many of the materials used are made by the pupils themselves, under the supervision of the teacher.

Very varied kinds of practical work are called for by polytechnical education, such as surveying, the use of calculating apparatus, the making of teaching materials (models and diagrams), the solution of problems (the determination of weight, the drawing up of estimates, the analysis of information collected during visits to industrial and agricultural undertakings), the study of machinery, etc.

The Ukrainian Ministry of Education prescribes the textbooks to be used for mathematics, and there are official recommendations as to how they should be used. Neither the prescriptions nor the recommendations are regarded as being in any way final; from time to time they are re-examined and revised in the light of current needs.

Teachers

Secondary teachers of mathematics receive their training in the physics-mathematics sections of the higher teacher-training institutes and the universities, the courses of which last respectively four and five years. After their final examinations, they are required to complete a certain period of practice teaching. The diploma syllabus comprises education, the history of education, psychology, school hygiene, the methodology of elementary mathematics, and appropriate practical work.

Their professional training includes two periods of practice teaching in a school, covering a study of teaching methods, and lasting six and eight weeks in the 3rd and the 4th year respectively of the training course.

The academic training of secondary teachers of mathematics covers mathematical analysis, the theory of the functions of variable quantities, higher algebra, the theory of numbers, theoretical arithmetic, analytical, differential, and descriptive geometry, technical drawing, the history of mathematics, a special course in elementary mathematics, and a special seminar.

No difficulty is experienced in recruiting secondary teachers of mathematics, as the country's needs in this as in other respects are planned for in advance. If they so desire, teachers may be appointed to suitable vacancies as soon as they qualify.

The opportunities of further training available to secondary teachers of mathematics comprise the following: (a) three-month courses, held each year, covering lectures, practical work, and an examination (teachers are required to attend these courses once every five years; lodging and maintenance is free); (b) consultations given by experts; (c) annual district or regional conferences at which teachers exchange experiences, the best reports being subsequently presented at national level, or published in book form if of outstanding interest; (d) lessons given publicly by leading teachers; (e) the mathematics committee formed in each school to draw up a teaching plan, solve special problems, draw up the attendance rota at lessons given by colleagues, etc.; (f) conferences and seminars organized for teachers by various specialist bodies to discuss questions of

theory and teaching methods and to demonstrate the use of calculating and measuring apparatus ; (g) the teachers' meetings held twice a year during the holidays, in the district centres and towns, for an exchange of ideas concerning ways of improving teaching ; (h) the educational journals published by the regional and central institutes, and the pamphlets on mathematics published by the " Soviet School " publishers.

Miscellaneous

Modifications in the mathematics syllabuses, made in 1954, are being gradually applied. They involve the elimination of topics of secondary importance and no special theoretical or practical interest, the introduction of topics of high practical value, especially in occupations in which measurement and calculation are important, and greater emphasis on interests and mental calculation, the use of the Russian calculating apparatus and the slide rule, the study of the metric system, and the calculation of areas and volumes.

The new syllabuses allot several hours to field work, and transfer some algebra materials from the 2nd to the 3rd year.

Other modifications to be gradually introduced in the near future involve the regrouping of certain topics in the algebra taken in the 3rd year, added stress on solid geometry in the 5th and the 6th year, the introduction of practical work in the 5th year, and of new elements of theory in the 6th year, and considerable lightening of the 6th year syllabus. The elimination of certain topics will generally be compensated for by the introduction of some concepts from higher mathematics, such as the derivate and its applications.

The modifications made in the mathematics syllabuses (and also textbooks) were preceded by investigations made by the institute of pedagogical research, the pedagogical institutes, the universities, the teachers' further-training institutes, and leading teachers.

Research generally bears on a given point, and is carried out by teams of experts on methods, on the basis of the experience, observations and suggestions of mathematics teachers. Every envisaged modification of a syllabus is subjected to thoroughgoing discussion by the teachers before it is brought into force, and every new textbook is tried out in a number of schools before being adopted.

Most of the textbooks and books on teaching or theory that have appeared in recent years in the Ukraine were published by the Ukrainian institute of pedagogical research, the Ukrainian " Soviet School " publishers, the Ukrainian teachers' further-training institutes, the pedagogical institutes, the universities, the school directorate of the Ukrainian Ministry of Education, and the correspondence-teaching methods office.

The new mathematics syllabuses will have been introduced in their entirety by the school year 1957-1958. The corresponding new textbooks are already being printed or prepared.

The documents enclosed with the Ukrainian reply to the International Bureau of Education questionnaire comprised the secondary mathematics syllabuses for 1955-1956, and the mathematics textbooks in current use in the Ukrainian secondary schools.

UNION OF BURMA

From the syllabuses sent by the Ministry of Education

Secondary Mathematics Syllabuses

Secondary education comprises a lower ("middle school") three-year cycle covering forms V to VII inclusive, the age range of the pupils being from 11 to 14 years, and an upper ("high school") two-year cycle covering forms VIII and IX, the age range of the pupils being 15 years and above.

Mathematics embraces arithmetic, algebra, geometry, and trigonometry.

Arithmetic is taught in the first three years. Elementary algebra, and geometry, are begun in the 1st year, and trigonometry in the 2nd year.

1st year (form V). Arithmetic: Local currency and measures, calculation of postage and charges for telegrams and trunk telephone calls, estimating monthly expenditure, the four operations, practical calculations. Algebra: introductory work. Geometry: the area of rectangles. — *2nd year (form VI).* Vulgar and decimal fractions, interest; mensuration and construction of simple solids; angles, triangles, polygons; the four algebraical operations. — *3rd year (form VII).* Time and distance, commercial transactions; equations with two unknowns, graphs, the properties of some geometrical figures. — *4th and 5th years (forms VIII and IX).* The use of logarithmic tables, the metric system, foreign currency and exchange, equations with three unknowns. Geometry: pyramids, cones, spheres, inscribed and circumscribed polygons, elementary trigonometry. Practical work.

Arithmetic thus covers the elementary rules, the Indian systems of measurement and currency, the metric system, proportions, square root, and interest.

Algebra embraces the basic formulas and equations with one or more unknowns.

The geometry course gives emphasis to the use of instruments and the properties of figures.

The theoretical aspects of trigonometry are illustrated by practical mensuration work.

UNION OF SOUTH AFRICA

From the reply sent by the Department of Education, Arts and Science

Separate treatment for each of the four provinces of South Africa having proved impracticable, the following information concerning the Transvaal only is offered as an example.

Transvaal

Place of Mathematics in Curriculum

Mathematics is taught throughout the five years of secondary education (standards VI to X). The age range of the pupils is from 14 to 18 years. The subject is an optional one, except for those pupils intending to continue their studies at university level.

The number of hours allocated to mathematics in each year, and the percentage they constitute of the total time-table, are as follows :

1st year		2nd year		3rd year		4th year		5th year	
Hours	%	Hours	%	Hours	%	Hours	%	Hours	%
1¼	5	4	16.6	4	16.6	5	20	5	20

Mathematics embraces geometry and algebra in the first three years, and geometry, trigonometry, algebra, and graphs in the last two years.

Mathematics is on an equal footing with all other subjects in promotion examinations. Special importance is accorded to the subject in the secondary school leaving examination, in the case of candidates intending to pursue their studies at higher level.

Aims of Mathematics Teaching

The aims of mathematics teaching given in official instructions are to give the pupils a knowledge of a definite sequence naturally arranged, and to develop their power of reasoning and of working out mathematical truths.

Syllabuses

1st, 2nd, and 3rd year (standards VI to VIII). Algebra : symbolical expression, solution of simultaneous equations of the first degree in two unknowns, problems, factors, graphs of linear functions. Geometry : theorems on straight lines parallel and intersecting, congruency of triangles, loci, constructions, problems. — *4th and 5th year (standards IX and X).*

Algebra : functional notation, remainder theorem, factors, simultaneous equations of the first and the second degree, quadratic equations, ratio and proportion, indices, logarithms. Graphs : straight line ; parabola, hyperbola, and circle ; dependent variable, independent variable ; constant function, tangent to a curve, gradient functions, determination, " h " and " D " method of notation, maximum and minimum values of functions up to the third degree. Trigonometry : trigonometrical ratios and relationships for angles up to four right-angles ; graphs, identities, solution of equations, properties and solution of triangles, easy problems of a practical nature in two and three dimensions. Geometry : theorems on concurrency, areas, and the circle ; loci and constructions ; problems, theorems and problems on ratio and proportion.

Methods of Teaching Mathematics

No official instructions are issued concerning the methods of teaching mathematics. The mathematics syllabuses, however, contain brief suggestions in this connection. The general principle stressed is the desirability of passing from the arithmetical to the algebraical concept, and from the practical to the theoretical. One of the functions of the inspectors, moreover, is to guide teachers and suggest methods.

At the transition stage between primary and secondary education, full use is made of the arithmetical knowledge the pupils have acquired at the primary stage, and the introductory course in mathematics covers the generalisation and abstraction of the arithmetical principles taught at primary level.

Correlation between the teaching of mathematics and that of other subjects is ensured by the teachers, who draw up detailed syllabuses making cross-references among the associated materials in the various syllabuses.

Applied mathematics as such is not included in the syllabuses, but questions of a practical nature are set. The setting of practical work as an introduction to formal teaching is a matter for the discretion of the individual teacher.

The choice of teaching materials and auxiliary aids is a matter for the discretion of the teachers ; schools are free to apply to the Department of Education for financial assistance in this connection. So far as textbooks are concerned, the Department has drawn up a list of approved textbooks, but has not issued any official instructions on the choice and use of textbooks ; under special circumstances textbooks other than those on the list can also be acquired.

Teachers

Secondary teachers of mathematics are required to possess the B.Sc. degree, obtained after a three-year university course in mathematics, and a teacher's certificate, obtained on completion of a fourth year of study at the university or a teacher-training school.

Increasing difficulty has been experienced in the last few years in recruiting teachers with a mathematics degree, as industry and commerce

absorb a large number of mathematics graduates. The Department is concerned about the situation, but has not so far taken any practical measures to meet it.

Secondary teachers have opportunities of further training through the lectures and refresher courses organized from time to time. As members of teachers' associations, mathematics teachers hold meetings to discuss teaching problems and methods. Similar meetings are held by the inspectors during their periodic visits. The educational journals publish articles on the teaching of mathematics.

Miscellaneous

No modifications are at present contemplated in the teaching of mathematics at secondary level. Modern developments in the mathematical field have not affected the mathematics syllabuses ; it is of interest to note, however, that differential calculus in connection with the gradient function in graphs was recently introduced.

Investigations on the teaching of mathematics has been mainly a matter of private initiative.

UNION OF SOVIET SOCIALIST REPUBLICS

From the reply sent by the Ministry of Education of the R.S.F.S.R.

Place of Mathematics in Curriculum

Mathematics is taught throughout the six years of secondary education (classes V to X of the ten-year schools). The age range of the pupils is from 11 to 17 years.

Six hours a week are allotted to mathematics in each year, representing an average of 20% of the total time-table.

The subject embraces arithmetic in the 1st and the 2nd year (classes V and VI), algebra and geometry in the 2nd to the 6th year inclusive (classes VI to X), and trigonometry in the 5th and the 6th year (classes IX and X).

Mathematics is a compulsory subject, so far as the school time-table is concerned. In addition, the subject is taken optionally in the school mathematics societies, for which higher education etablishments organize systematic courses. Popular scientific works, moreover, are published for these societies, dealing with a wider range of topics than the school syllabuses. Pupils also participate in competitions called " mathematical olympiads ", which involve the solution of more difficult problems than those they take in class.

Mathematics ranks as a major subject. In each year there is an examination (oral and written in the 4th, the 5th and the 6th year) in the subject.

Aims of Mathematics Teaching

The aims of mathematics teaching are given in the official syllabuses as follows : to impart elementary knowledge in arithmetic, algebra, geometry and trigonometry ; to teach and accustom the pupils to apply mathematics to the solution of various practical tasks ; to contribute to the development of the pupils' power to think logically, and of their comprehension of space ; to contribute to the pupils' general and polytechnical education ; to prepare them for an occupation and for study at higher level.

Syllabuses

1st year (class V). The beginning of systematic arithmetic (whole numbers, vulgar and decimal fractions). Elementary geometry. — *2nd year* (class VI). Arithmetic : percentages, proportions and proportional values. Algebra : algebraical expressions, positive and negative numbers, operations, the solution of simple equations of the first degree in one unknown with the help of the properties of the arithmetical operations.

Geometry : The beginning of systematic geometry ; elementary concepts (the straight line, angles, the circumference), parallelism, triangles. — *3rd year* (class VI). Algebra : the resolution of polynomials into multipliers, algebraical fractions, equations of the first degree in one unknown, systems of equations of the first degree in two or three unknowns, introduction to the rectangular system of coordinates with graphs of the functions. Geometry : quadrilaterals (parallelograms and trapezia), the circumference, inscribed and circumscribed triangles. — *4th year* (class VIII). Algebra : degrees and roots, equations of the second degree, the square of a trinomial, systems of equations of the second degree. Geometry : ratios and proportionality of segments, homology and similarity, dimensional relationships of the elements of the triangle and the circle, the trigonometrical functions of an acute angle and the solution of right-angled triangles, measurement of the areas of polygons. — *5th year* (class IX). Algebra : limits, progressions, demonstrative and logarithmic functions, logarithms. Geometry : regular polygons, the circumference and the area of a circle, solid geometry (as far as polyhedrons). Trigonometry : the trigonometrical functions of any angle, the algebraical relationships between the trigonometrical functions of a given angle, conversion formulas, the trigonometrical functions of the numerical proof, the theorem of addition and its corrolaries. — *6th year* (class X). Algebra : the functions and their analysis, the derivate and simple applications, complex numbers, inequalities, the analysis of equations and systems of equations. Geometry : polyhedrons, solids of revolution. Trigonometry : the solution of triangles.

Some reference is made to the history of mathematics. Pupils are required to know the part played by mathematics in the history of civilization, its cultural value, its place in the study of other scientific subjects and in the realization of the objectives of socialism, and the lives of the great mathematicians. An introduction to the principles of axioms is given in the geometry course in the 6th year.

Methods of Teaching Mathematics

Instructions concerning the organization and methods of mathematics teaching may be found in the mathematics syllabuses and in the circular on methods (" The Teaching of Mathematics in Classes V to X in Secondary Schools ") issued by the school directorate of the Ministry of Education. It is there stated that no one method can be regarded as being of universal validity. In giving a systematic course the teacher must choose methods conforming to the age of the pupils and the topic under discussion. Two methods commonly employed, especially in the lower years, are the heuristic method, fostering reasoning, and the discussion method. In the course of the work the pupils learn to do various exercises systematically : the solution of problems, the making of models, etc. In the upper years stress is laid on formal exposition by the teacher, in the form of a short lecture, followed by practical work.

In accordance with the principles of teaching now in force, what is regarded as important is not so much the scope of the syllabus, as the extent to which the topics dealt with have been learned and assimilated. The results obtained are satisfactory ; on leaving the secondary school

the pupils are well able to follow the instruction given in the higher technical schools. Considerable importance is attached to the work the pupils do outside class hours in the school mathematics societies and evenings.

There are instructions concerning the teaching methods to be used in classes IV and V ; in class IV the class teacher takes all the subjects, while in class V, for the first time, the various subjects, including mathematics, are taught by specialist teachers.

The teaching of mathematics is correlated with that of art, physics, chemistry, and economic geography.

Schools are provided with teaching materials at State expense, in accordance with a list of requirements drawn up by the Ministry of Education. The materials for mathematics include models of geometrical solids, charts, apparatus and instruments necessary for practical work (set-squares, astrolabes, clinometers), and films (on the trigonometrical functions, the surfaces of revolution, etc.).

In accordance with requirements of polytechnical education, practical work, whose nature and extent varies according to the year, forms a compulsory part of the mathematics syllabuses. A certain number of hours per week are allotted to surveying, the making of models, drawings and diagrams, and calculations with the help of calculating machines and slide-rules. Depending on the nature of the topic being dealt with, the age of the pupils, and the requirements of the syllabus, such work is done either before the teacher's formal exposition, or after it, when the theory taken is being applied.

The new syllabuses drawn up by the Ministry of Education of the R.S.F.S.R. and first introduced into the schools in 1954-1955, make provision for handicrafts in classes I to IV, for workshop or farm activities in classes V to VII, and for practical exercises in mechanics, agriculture and electro-technics in classes VII to X. In this way the pupils are enabled to assimilate the theory of mathematics and to apply their knowledge to the solution of practical problems.

Textbooks are approved by the Ministry of Education, and must be used in all schools. The Ministry issues special circulars instructing teachers in the use of textbooks. The textbooks are published by the State educational publishing house attached to the Ministry of Education, in sufficient quantity to ensure their distribution to all pupils.

Teachers

Secondary teachers of mathematics are trained in the physics and mathematics faculties of the universities and the pedagogical institutes, and must hold a leaving certificate awarded by an appropriate educational establishment.

Their scientific training is adequate ; they study for four years at a pedagogical institute and for five years at a university, covering basic courses in higher mathematics. Their professional training includes psychology, pedagogy, the history of education, the methodology of mathematics, and periods of practice teaching in demonstration schools under the supervision of methodology specialists.

No difficulty is experienced in recruiting secondary teachers of mathematics, as the school requirements in this respect are planned for in advance. In recent years some localities lacked such teachers, on account of the rapid increase in secondary enrolments; as a temporary measure, the education authorities authorized physics teachers to take mathematics, and mathematics teachers were allowed to teach in two schools at once. The output of graduates by the higher establishments is now sufficient to meet all requirements in respect of secondary teachers of mathematics.

The mathematics teachers of each school hold special meetings to discuss ways and means of improving their work, to give reports on teaching methods, to examine the results obtained by their pupils, etc.

A pedagogical centre, moreover, exists in each of the district centres, at which teachers can get the books they need, and advice on teaching methods from experienced teachers.

Teachers' further-training institutes exist in each of the regional centres, offering advice, special seminars, lectures, and practical exercises.

"Pedagogical readings" are organized annually by the further training institutes and the R.S.F.S.R. Academy of Education, at which teachers can give reports on original work they have done on teaching methods.

Teachers can also obtain advice on teaching methods from the professors of the universities and the pedagogical institutes.

The Moscow Mathematics Society has a section for secondary teachers of mathematics. Leading Soviet mathematicians participate in the work of this section; during its meetings all the problems of mathematics teaching are discussed, and reports are presented on the present position in the field of mathematics.

A special journal ("Mathematics at School") is published for secondary teachers of mathematics. It appears six times a year, in an edition of 90,000 copies, and is available in every school library and by subscription. Each issue contains 10 to 12 pages.

In order to raise the professional standard of teachers, moreover, generous and widespread distribution is made of academic and popular works covering the history, the application, and the methodology of mathematics. These works are published by the school publishing house of the R.S.F.S.R. Ministry of Education, by the publishing house of the Academy of Educational Science, the U.S.S.R. Academy of Science, the State technical publishing houses, etc.

Miscellaneous

In conformity with the government's decision to introduce universal ten-year education and polytechnical education, the mathematics syllabuses were revised. The new syllabuses were placed on trial in 1954-1955, and in the light of the results they give, they will be completed and improved.

Modern developments in the mathematical field are exerting an indirect but undoubted influence on secondary syllabuses and textbooks, the approach to the fundamental notions, and the content of the optional courses offered in the school societies and the olympiads, and also on the

secondary teacher training offered by the universities and the pedagogical institutes. Such influence is brought to bear through the work done in the scientific and the methodological field by mathematicians working in the Ministry of Education, the Academy of Educational Science, the teachers' further training institutes, and other establishments.

Research and experiments on the teaching of mathematics are now being carried out at the demonstration schools attached to the pedagogical institutes. New syllabuses and teaching methods are tried out at the special schools attached to the R.S.F.S.R. Academy of Educational Science. Textbooks are also placed on trial before their final adoption.

In addition to the textbooks referred to above, the school publishing house of the R.S.F.S.R. Ministry of Education has also published works on the teaching of mathematics in secondary schools.

The new arithmetic and algebra syllabuses will be introduced in the 2nd year (class VI) at the beginning of 1955-1956. The change-over to the new syllabuses will be completed in 1957-1958 ; new textbooks are being drawn up, and will be published shortly.

UNITED KINGDOM

England and Wales

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

There are three different types of secondary schools: (a) grammar schools, with a seven-year course, the age range of the pupils being 11 to 18 years; (b) technical schools, with a five-year course, the age range of the pupils being from 11 to 16; (c) modern schools, with a four-year course, the age range of the pupils being from 11 to 15. Mention should also be made of the independent "public schools" for boys from 13 to 18 years of age, and the independent schools for girls, which usually follow the grammar schools as regards age range, but are less highly selective.

About one seventh or one eighth of the total time-table is devoted to mathematics by all pupils under the age of 16. Mathematics is rarely optional before the age of 16, but in some grammar schools pupils not gifted mathematically are sometimes allowed to drop the subject or to take a simplified course at about the age of 14. From 16 to 18 years the subject is optional.

No special importance is as a rule attached to mathematics, as compared with other subjects in the secondary curriculum.

Aims of Mathematics Teaching

There are no official instructions concerning the aims of mathematics teaching at secondary level. Head teachers of schools are generally free to frame their own schemes of work and to decide such matters as teaching methods, textbooks, educational aims, etc.

Syllabuses

As already noted, syllabuses in any subject are generally drawn up, not by any official body, but by the head teachers of schools. In grammar schools and technical schools (and the corresponding classes of independent schools), however, the mathematics syllabuses will generally be found to cover the following:

1st year. Arithmetic, simple formulas and equations, intuitive geometry. — *2nd year.* Arithmetic, further algebra, beginning of formal geometry. — *3rd year.* Arithmetic, numerical trigonometry, Pythagoras's theorem. — *4th year.* Mensuration, logarithms, quadratic equations. — *5th year.* Civic arithmetic, gradient of graph, area under graph, systematic geometry.

In the last two years of grammar and independent schools, the pupils who decide to take mathematics are in two categories : (a) those who also take science or some other subject ; (b) those who specialize in mathematics, taking at most one other subject (usually physics). The mathematics syllabuses for category (a) pupils are the advanced level syllabuses for the " General Certificate of Education " examinations, which cover both pure mathematics and applied mathematics (kinematics, dynamics, statics and hydrostatics). The mathematics syllabuses for category (b) pupils are the scholarship level syllabuses for those examinations, which include and go beyond the advanced level syllabuses.

The mathematics syllabuses for modern schools cover mainly arithmetic, mensuration, and very simple algebra.

Some schools include the history of mathematics and an introduction to axioms.

Methods of Teaching Mathematics

There are no official instructions concerning methods of teaching mathematics, but pamphlets of suggestions have been published by the Board of Education. The reports published for the Mathematical Association are the chief guides in this connection.

There are no official instructions, moreover, concerning the transition from primary to secondary education. As to the correlation of mathematics with other subjects, this is effected in each school by discussion between the teachers involved. As indicated above, applied mathematics is not normally taken before the advanced stage ; most pupils do some experimental mechanics as part of a science course ; about 3% of the pupils in grammar schools and about 30% in the " public schools " take additional mathematics, which includes mechanics, before entering the 6th year ; a few schools study statistics.

Surveying is often used as an introduction to geometry or trigonometry.

The teaching materials used in mathematics lessons are chosen by the individual teacher. There are no official instructions concerning the use of textbooks.

Teachers

Professional training for secondary teachers of mathematics is not compulsory, but is encouraged. Teachers in publicly maintained schools, however, must be qualified teachers. A teacher is regarded as qualified if he has satisfactorily completed an approved course of teacher training, or holds an advanced qualification such as a university degree.

Some difficulty is experienced in recruiting secondary teachers of mathematics. There is a demand for mathematicians in industry, which offers higher salaries and better prospects of promotion. The salaries for teachers of advanced work were raised recently.

Serving teachers of mathematics have opportunities of securing further training through the numerous courses, lasting from one to ten days, arranged by the Ministry of Education, the universities, and the local education authorities. The Mathematical Association is largely an association of teachers of mathematics.

Miscellaneous

The main modifications envisaged (and already established in about half of the grammar schools) in the teaching of mathematics at secondary level are the reduction of formal geometry and algebra, and the earlier use of trigonometry and the idea of functionality, leading to the introduction of the calculus.

The freedom enjoyed by the schools from official direction implies that every school syllabus is itself experimental.

Northern Ireland

From the reply sent by the Ministry of Education

Place of Mathematics in Curriculum

There are three types of secondary schools in Northern Ireland: grammar schools, providing a general education course up to university entrance level; secondary intermediate schools, providing a general education course with a practical bias, and technical intermediate schools, conducted in association with an institution of further education, and providing pre-vocational courses for boys who have completed a two-year general course at a secondary intermediate school.

Mathematics is taught throughout the seven years (pupils of 11 to 18 years old) of the grammar schools, and the four years (pupils of 11 to 15 years old) in the secondary intermediate schools, and at all stages (pupils of 13 to 16 years old) in the technical intermediate schools. The amount of time allocated to the subject varies somewhat from one school to another, but averages about five hours a week, which in the three types of schools represents respectively roughly 20%, 15%, and 20% of the total time-table.

In the upper division of the grammar schools, moreover, selected pupils take advanced mathematics, to which additional time is devoted varying in amount from school to school and also according to the number of branches of the subject studied.

In the lower division (pupils 11 to 14-15) of the grammar schools, mathematics comprises arithmetic, algebra, geometry, and trigonometry. In the upper division (pupil 14-15 to 18), three mathematical courses are available: the traditional course, covering the same four branches as in the lower division; the alternative course, covering the same four branches, and calculus in addition; the advanced optional course, covering the same four branches, and calculus and mechanics in addition.

Candidates for the Junior Certificate (taken at the age of 14 plus, after a three-year or four-year course) must present two of the following four branches: arithmetic, algebra, geometry, trigonometry. Candidates for the Senior Certificate must pass in one mathematical subject or one scientific subject (physics, biology, geography, domestic science, etc.).

In the secondary intermediate schools, where only a few children remain after the compulsory school age, mathematics embraces arithmetic in all years, algebra and geometry for the abler pupils, and some elementary trigonometry for the few really able pupils.

In the technical intermediate schools (pupils 13 to 15 years in general), mathematics embraces arithmetic, algebra, geometry, and trigonometry in the 1st and the 2nd year, and the same branches, together with elementary differential and integral calculus and informal solid geometry, in the 3rd year. Mathematics and science are the most important subjects in these schools. The time allocation for mathematics varies from school to school.

Aims of Mathematics Teaching

There are no official instructions concerning the aims of mathematics teaching for any of the three types of secondary schools. In general it may be said that the grammar school courses are aimed at university entrance, those of the secondary intermediate schools at a certain minimum of general education, and those of the technical intermediate schools at preparing the boys for entry to skilled apprenticeships and for further technical studies.

Syllabuses

As already noted, the mathematics syllabuses of the grammar schools cover arithmetic, algebra, geometry, and trigonometry, in the lower division, and, in the upper division, a traditional course (the same four branches), an alternative course (the same four branches, and calculus in addition), and an advanced course (the same four branches, and calculus and mechanics in addition).

The mathematics syllabuses for the technical intermediate schools are similar, except that they give less emphasis to theory and more emphasis to practical applications.

There are no set syllabuses for the secondary intermediate schools. Each school designs its own syllabuses to suit the needs and interests of its pupils.

The history of mathematics is not included in the syllabuses of any of the three types of secondary schools. The library of many grammar schools will contain books on the subject.

Methods of Teaching Mathematics

There are no official instructions or suggestions concerning methods of teaching mathematics at secondary level. The commonest method is that of class instruction; the teacher discusses the new topic with the class, making suitable use of the blackboard; the pupils then work at an assignment of examples from their textbook, with or without the assistance of the teacher. Some project work is undertaken in the secondary intermediate schools.

No formal provision is made in regard to teaching methods at the transition stage from primary to secondary education. Teaching at primary level aimed directly at the transfer examination, is officially discouraged.

The correlation of mathematics with other subjects is encouraged, but it is largely left to the individual school to effect. The other subjects particularly involved are science, geography, crafts, and mechanical drawing.

The teaching materials and auxiliary aids used for mathematics lessons include filmstrips (especially in the secondary intermediate schools), and a variety of two-dimensional and three-dimensional models.

Practical work plays little part, except for the advanced course in the grammar schools. Activity methods are little used in either grammar schools or technical intermediate schools, but are extensively used in the secondary intermediate schools.

The schools are free to select their own textbooks. The inspectors are often consulted in this connection. The secondary intermediate schools submit lists of proposed textbooks to the Ministry of Education and receive official approval.

Teachers

Teachers of mathematics in the grammar and the technical intermediate schools are required to hold at least a degree with two courses in mathematics; pedagogical training is not essential; for the technical intermediate schools, scientific or industrial experience is an additional qualification. Teachers of mathematics in the secondary intermediate schools must hold a similar degree, or a degree and an education diploma, or must have completed a three-year course of pedagogical training.

So far as the recruitment of mathematics teachers is concerned, the position is improving in the grammar schools, although many graduates take up industrial or university appointments; no special measures have been taken to remedy the situation. No difficulty is experienced in recruiting mathematics teachers for the secondary intermediate schools. In the technical intermediate schools, similar difficulty is met with in this connection as in the grammar schools, and a special committee has been set up to consider the problem.

Refresher courses are held periodically by the Ministry of Education, and teachers may attend courses and seminars held elsewhere in the United Kingdom.

Miscellaneous

No modifications are envisaged in the mathematics syllabuses of any of the three types of schools, except that new syllabuses for the advanced course of the grammar schools are to be introduced in 1957.

Experimental work for the teaching of mathematics, on standardized tests, is being conducted by the education department of Queen's University, Belfast. No books have been recently published on the teaching of mathematics; the report for 1951-1952 of the Ministry of Education contained a section on the matter.

The trend of mathematical teaching is greatly influenced by developments in Great Britain. Lecturers from England and Scotland come to Northern Ireland, teacher exchanges are made with English schools, and some Northern Ireland students aim at entrance to universities in England. Individual secondary intermediate schools have successfully introduced trigonometry, logarithms, and elementary surveying; such experiments are encouraged by the Ministry of Education.

Enrolment in the technical intermediate schools is selective, about 1,000 boys being admitted per year. The courses in mathematics in these schools, as already noted, are comparable to the traditional and alternative (ordinary level) courses in the grammar schools, though the treatment is different, much more emphasis being put on practical applications. The courses are influenced, moreover, by the requirements of the Institution of Electrical and Mechanical Engineers.

Scotland

From the reply sent by the Scottish Education Department

Place of Mathematics in Curriculum

On promotion to a secondary school, pupils are allocated, according to ability, either to a three-year course or to a five-year course. The three-year course caters for pupils who intend merely to complete their period of compulsory schooling, the age range of the pupils being from 12 to 15. The five-year course is for pupils preparing to sit for either the lower or the higher grade of the Scottish Leaving Certificate Examination, and the age range of the pupils is from 12 to 17.

Mathematics appears in the curricula under that name. It is a required subject throughout the three-year course, under three forms: a general course for boys and girls of average ability; a course mainly in arithmetic for girls of average ability; a very simple course for boys and girls of low ability. In the five-year course, it is a required subject in the first three years; in the 4th and the 5th year, the great majority of pupils profess mathematics, but a few do not require mathematics at this stage.

The time allocated to mathematics varies a little from school to school, but in general consists of six forty-minute periods a week (four forty-minute periods in the case of pupils taking either of the simplified courses) in the first three years, and normally seven periods a week in the 4th and the 5th year.

It should also be noted that a proportion of the pupils who pass the higher grade of the Scottish Leaving Certificate Examination, at the end of the 5th year, elect to continue the study of mathematics, and for them three additional courses are provided: elementary analysis, geometry, and dynamics (applied mathematics). Normally these courses are all begun in an additional, 6th year, but in a few schools dynamics is begun in the 4th or the 5th year.

So far as the relative importance of mathematics is concerned, it is to be noted that arithmetic is normally an important factor in the promotion of pupils from primary to secondary education. Within the secondary school, however, mathematics has no special significance in regard to promotion. In regard to the Scottish Leaving Certificate Examination, lower and higher grade passes in mathematics have the same standing as the corresponding passes in other subjects; mathematics is the only subject, however, in which papers of a yet more advanced standard are provided.

Aims of Mathematics Teaching

The aims of mathematics teaching in the first three years of the secondary school course are set out in "Junior Secondary Education", published by the Scottish Department of Education. Firstly, there are utilitarian aims, of which the following is a summary. "Some ability to cope with elementary calculation and some familiarity with weighing, measuring, and estimating are essential for the ordinary person in everyday life. There is therefore a basic minimum of mathematics, of purely utilitarian value, which must be included in the education of every pupil. Further, both his school life and his home life bring him into touch with a wide range of simple applications of mathematics ... Again, the mathematics course should help to prepare the pupil for his responsibilities as a citizen. If he is to take an intelligent interest in the issues of the day he should understand the arithmetical principles involved in rates, taxes, tariffs, and duties. He should also be able to read and appraise graphical representations and statistical statements, since these are now being widely used to inform the general public on matters of importance". The memorandum goes on to state: "The school must also provide the pupil with any basic knowledge required for the successful pursuit of his future employment. This means that the teacher should be aware of the mathematical techniques called for in the main occupations of the neighbourhood, and that the course should give attention to these techniques and to the underlying mathematical principles. It will not be possible, and may not even be desirable, to prepare pupils for all the specific mathematical needs of a particular occupation, but, in drawing up courses for boys, regard should be had, among other factors, to the main mathematical ideas and techniques that serve the general needs of technology. The teacher should also make sure that the course takes some account of the mathematics in further education courses which his pupils are likely to follow when they take up employment". In addition to these utilitarian objectives, the memorandum also mentions aims of an educational or intellectual character, such as developing the pupils' powers of observation and reasoning, and insistence on clarity of expression. "The teacher should also try to show his pupils the part which mathematics has played in the development of civilization and help them to appreciate the thought behind the technical achievements of the modern world. Again, although in the junior secondary school there is no place for a formal course in mathematics or for an abstract treatment of the subject, this does not mean that pupils should not on occasion be given a glimpse of mathematical relationships". Finally the memorandum adds; "Mathematics is more

than a mere accumulation of technical knowledge ; it is a mode of thought, and the teacher should try to afford his pupils an opportunity of sharing in this kind of thinking, if only in a very simple and elementary way ".

The aim of mathematics teaching in the five-year course is to provide pupils with the basic mathematics they are likely to require either in their vocation or as a foundation for advanced study.

Syllabuses

The official mathematics syllabuses for the first three years, i.e., the junior secondary school course, are to be found in " Mathematics in Secondary Schools ", published by the Scottish Education Department. They leave considerable room for adaptation to the varying aptitudes and capacities of the pupils. Five different syllabuses, in fact, are offered ; the general course (" Syllabus III "), for boys and girls of average ability, may be summarized as follows :

Arithmetic : elementary rules, money, common weights and measures, vulgar fractions, decimals, metric system, proportion (simple and compound, direct and inverse), proportional division, square root, percentages, discount, profit and loss, simple interest, use of ready reckoners, rates, taxes, insurance, calculation of areas and volumes, applications of the theorem of Pythagoras and converse. Algebra : problems involving simple and simultaneous equations, use of simple formulas, statistical graphs, construction and use of graphical ready reckoners to find, for example, prices. Geometry : use of instruments (ruler, set-square, compasses, protractor), construction of geometrical plane figures and consideration of minimum data required, properties of triangles, polygons, and the circle.

The official mathematics syllabuses for the five-year course leading up to the Scottish Leaving Certificate Examination are also to be found in " Mathematics in Secondary Schools ". The programme for the first three years (" Syllabus I ") covers geometry, algebra, arithmetic, and some trigonometry, and that for the 4th and the 5th year embraces further study of these four branches and, for the higher grade of the Examination, coordinate geometry and calculus.

None of the syllabuses normally include the history of mathematics, or any introduction to axioms as such.

Methods of Teaching Mathematics

There are no official instructions as such concerning methods of teaching mathematics at secondary level, but a number of suggestions in this connection are contained in the two volumes already referred to, " Mathematics in Secondary Schools ", and " Junior Secondary Education ". These suggestions stress the need for a practical emphasis, and a choice of topical subjects and simple examples.

In regard to the transition from primary to secondary education, it is to be noted that there is a minimum content in arithmetic for the primary course, and secondary teachers are normally aware of what their pupils have covered in this connection. The correlation between mathematics and other subjects is usually established by arrangements among the teachers in each individual school.

Classes in applied mathematics are available for five-year pupils wishing to take the additional paper of the Scottish Leaving Certificate Examination in dynamics ; other pupils receive some instruction in applied mathematics in their courses in science and technical subjects.

In respect of activity methods, it may be said that official encouragement has been given to their use, but they have not yet been widely adopted.

The teaching materials and auxiliary aids used for mathematics lessons generally include mathematical models, and instruments such as simple theodolites (often made in school), surveyor's chains, and compasses. Films or filmstrips are rarely used.

There are no official instructions concerning the choice and use of textbooks. In each school the principal teacher of the subject normally selects the particular textbooks to be used, subject to more or less routine approval by the headmaster and by the education authority.

Teachers

Secondary teachers of mathematics are of two types : (a) holders of the " Teacher's Special Certificate in Mathematics ", eligible to teach at all levels ; (b) sub-specialist teachers of mathematics, eligible to teach the subject in the lower years.

To obtain the " Certificate ", teachers of type (a) are required to hold a Scottish university first or second class honours degree in mathematics, or the equivalent, and to have completed successfully a one-year course of teacher training. Teachers of type (b) are required to hold a university degree which includes passes on the ordinary and higher standards, and to have successfully completed a one-year course of teacher training.

The report [1] of a special committee was recently published on the recruitment of mathematics and science teachers. The great difficulty experienced in this connection is ascribed in this report to (a) the increase in the school population and the consequent increased demand for teachers, resulting from the prolongation of compulsory education in 1947, (b) the increased emphasis on science subjects after the second world war, in consequence of the added importance of science, (c) the increased demand for mathematicians and scientists in industry and other branches of production, (d) the preference shown by young graduates, in a period of full employment, for research.

The same report makes a number of recommendations for remedying the situation, including short-term measures such as the appointment of unqualified or semi-qualified teachers and of qualified teachers in or near retirement, the grant of scholarships to honours graduates to cover the one-year course of teacher training , and the deferment of military service for specialist teachers for an indefinite period, and long-term measures comprising mainly added facilities for studying and an increase of specialist teachers' salaries. Some of the proposed recommendations are

[1] Scottish Education Department : " Supply of Teachers of Mathematics and Science in Scotland ". Edinburgh, Her Majesty's Stationery Office, 1955.

already being put into practice. Classes for the further instruction of teachers, moreover, are conducted throughout the school year and in the summer vacation; those relating to mathematics are usually on the methods of teaching the subject, and are designed to enable in-service teachers to become sub-specialist teachers of the subject. Efforts have also been made to stimulate recruitment to the teaching profession generally by issuing appropriate posters and pamphlets.

In regard to further training, it is to be noted that there is comparatively little interchange of teachers, that refresher courses are occasionally organized on methodology, and that many teachers are members of the Edinburgh Mathematical Society and the Glasgow Mathematical Association. There is also an English publication, The Mathematical Gazette, which is extensively read in Scotland.

Miscellaneous

It has been decided to modify the lower grade of the Scottish Leaving Certificate Examination, in order to make it suitable for a candidate in the 4th year of the secondary course.

Modern developments in the mathematical field have had some influence on secondary education. The increasing emphasis in modern mathematics on the analytical aspects, for example, has been accompanied by a similar trend in the schools, and the importance of formal geometry relative to the other branches is gradually diminishing.

Little or no experimental work on the teaching of mathematics is being carried out at present.

UNITED STATES

From the reply sent by the Office of Education

Place of Mathematics in Curriculum

Grades seven through twelve are considered the secondary school level. The age range of the pupils is from 11 or 12 to 17 or 18. In most states, mathematics is required of all students in grades seven through nine. In grades ten to twelve, mathematics is usually not required but it is elected by many students. For example, a survey of randomly selected high schools in 1952 indicated that approximately one third of the students in the tenth grade were enrolled in mathematics and a little more than one-tenth of the students in the twelfth grade were enrolled in a mathematics course.

In grades seven and eight approximately one sixth of the pupil's time will be devoted to the study of mathematics. In grade nine, one fourth to one fifth of his time will be devoted to mathematics. In grades ten to twelve mathematics may be elected, in which case approximately one fourth of his time is devoted to that subject. All mathematics classes usually meet 45-55 minutes each day, five days a week for 180 days during the year.

All mathematics in grades seven and eight is entitled general mathematics or arithmetic. In grade nine algebra or general mathematics is studied. Plane geometry is usually taught in the tenth grade ; intermediate algebra in the eleventh ; and solid geometry and plane trigonometry in the twelfth year.

The same importance is attached to mathematics as to other subjects in the secondary curriculum.

Aims of Mathematics Teaching

Each state in the United States is responsible for its own educational system. Each State in turn delegates much of this responsibility to large city school systems. Many of these school systems have outlined their aims for the teaching of mathematics ; however, there is no syllabus or course of study or list of objectives that have been issued by a federal government agency. Most states issue only general guides and local schools are free to develop the offerings consistent with local needs. The newer guides or courses of study suggest that the emphasis of mathematics education should be on understanding of the mathematical concepts as well as the ability to compute, desirable attitudes toward mathematics, and the ability to collect, organize, and interpret data, and express a generalization in mathematical symbols.

Syllabuses

There is no syllabus of mathematics prepared on a national level to be used by all schools. Perhaps the textbooks are the best single index of the content being taught in the schools. The National Council of Teachers of Mathematics has published a yearbook, " The Place of Mathematics in Secondary Education, 15th Yearbook ". This yearbook proposed content for high school mathematics courses which is reflected in many new textbooks.

The history of mathematics is usually introduced as the different topics are studied and not as a special unit. Introduction to axioms usually occurs in the teaching of plane geometry ; however, a few teachers introduce the concept in the study of algebra.

Methods of Teaching Mathematics

There are no official instructions or suggestions concerning the methods to be used in teaching mathematics in secondary schools. However, national professional organizations such as the National Council of Teachers of Mathematics suggest that more consideration be given the talented pupil through such methods as individual instruction, small group work within the class, and assignment of supplementary material. The National Council of Teachers of Mathematics has published a yearbook entitled " Emerging Practices in Teaching of Mathematics " which presents some of the more promising practices.

A brief explanation by the teacher of the mathematical principle is usually followed by a discussion of sample problems and exercises, and perhaps a third of the period will be devoted to supervised study. There is some tendency to develop mathematical concepts and principles inductively and to lead the pupil to the discovery and formulation of these principles for himself.

The colleges of education provide instruction on teaching methods for mathematics at the transfer stage from primary to secondary school. The federal government does not provide instruction in a national syllabus.

The coordination established between mathematics and other subjects taught in secondary schools varies greatly among the schools. The greatest medium of coordination is in the applied problem in mathematics. Many of the newer textbooks include problems related to physics, economics, etc.

The Office of Education published a circular entitled " Teaching Materials for Mathematics Classes " which contains a list of free and inexpensive aids.

A regional magazine, the "Chicago Schools Journal", recently published a supplement which gave lists of films, filmstrips, models, and devices that may be secured by teachers. Many city school systems and state departments of education provide films, filmstrips, and models that may be used in the teaching of mathematics. The U.S. Office of Education does not provide this material.

Activity methods in connection with the practical exercises preceding formal teaching are used to a small extent and mostly with the slow learners.

Each state department of education usually determines a list of recommended books. From this list the school staff may select the book that will most nearly fit their needs. In the relatively few cases where textbooks are not used, pupils are guided in the definition and formulation of concepts and principles developed through inductive processes and in organizing these materials in their own notebooks.

Teachers

The certification requirement of teachers of mathematics in the secondary schools is determined by each state. In most cases the teacher of mathematics will have a four-year college diploma (a bachelor's degree) and will have studied college mathematics at least through analytic geometry. In some cases they will have majored in mathematics. A major is usually about one fourth of the entire courses taken in colleges.

There is difficulty in recruiting mathematics teachers. Some of the reasons are low salary scale, lack of prestige, and the cost of the long period of education. National organizations have sponsored campaigns to increase the salary scale of teachers, to motivate pupils to select teaching as a career, and to show the importance of teaching.

The means employed to keep serving teachers of mathematics up-to-date in their knowledge include the interchange of teachers with other countries, special courses, and workshops during the summer, aided by national foundations. Industry and professional organizations have provided scholarships for short refresher courses. Many large school systems plan an in-service education programme which will include monthly conferences of teachers. National professional organizations have several each year for mathematics teachers.

Miscellaneous

There seems to be an indication that greater emphasis will be placed on the meanings of mathematics and less on manipulative procedures; also that more stress will be laid on better mathematics opportunities for the superior pupil.

Modern mathematics has had some effect on secondary school mathematics. The National Council of Teachers of Mathematics plans a yearbook on this subject. Teachers of mathematics in the 13th and 14th grades (first two years of college—students approximately twenty years old) indicate that modern mathematics is affecting the offerings in these grades.

Experiments and studies are usually carried out under the direction of universities or departments of education. In some cases the mathematics department in a large school system will promote a high school study.

VIETNAM

From the reply sent by the Ministry of National Education

Place of Mathematics in Curriculum

The duration of secondary education is seven years. The teaching of mathematics begins in the 2nd year, the *classe de sixième* (pupils of 12 or 13 years of age), and continues up to the final year, the *classe de première* (pupils of 17 or 18 years of age).

The weekly time allocation for mathematics, out of a total of twenty-nine hours a week, is as follows:

2nd year		3rd year		4th year		5th year		6th year		7th year	
Hours	%	Hours	%	Hours	%	Hours	%	Hours	%	Hours	%
3	10.34	3½	12	3½	12	6	20.68	6	20.68	8	27.5

The hours indicated above for the 5th, the 6th and the 7th year (the upper cycle) are those of the mathematics section; the time allocation for mathematics is less in the literature and experimental science sections.

Mathematics is officially designated as follows: *2nd and 3rd year:* arithmetic, algebra, plane geometry, geometrical drawing. — *4th year:* algebra, spatial geometry, geometrical drawing. — *5th year:* algebra, plane geometry. — *6th year:* algebra, trigonometry, solid geometry. — *7th year:* arithmetic, algebra, trigonometry, mechanics, additional geometry, cosmography.

Mathematics is regarded as a specially important subject, and has a high maximum mark in the secondary school leaving examination.

Aims of Mathematics Teaching

Mathematics at secondary level forms part of the pupils' general education, and aims at the development of their mind and reasoning power. The subject also serves as preparation for higher studies in the case of pupils who intend to become teachers, engineers and technicians, or to take up a post in industry.

Syllabuses

2nd year. Operations with whole numbers and decimals; exercises. Geometry: straight lines, areas, angles, triangles, similar triangles, polygons, properties, geometrical drawing. — *3rd year.* Lowest common multiple, highest common factor, percentages, introduction to algebra, first-degree equations. Geometry: triangles and theorems on them; theorem of Thales, polygons, exercises. — *4th year.* First-degree equations

with two unknowns, second-degree equations. Geometry : revision, introduction to trigonometry, introduction to solid geometry, area of polyhedrons and spheres. — *5th year*. First-degree and second degree equations, functions, logarithms, trigonometry. — *6th year*. Study of functions. Geometry : revision of plane and solid geometry, trigonometrical functions. — *7th year*. Arithmetic, algebra, trigonometry, additional geometry, cosmography.

The syllabuses do not include the history of mathematics or the principles of axioms.

Methods of Teaching Mathematics

The usual method of teaching mathematics is the intuitive one. Coordination between the teaching at primary and at secondary level is ensured through a thorough revision in the 1st year, of the work the pupils have done at the primary level.

The mathematical knowledge required for the study of physics is always taught in advance, at the suitable time, and coordination between the two subjects is thus ensured.

Aids for the teaching of mathematics include wall charts, geometrical figures, and slides. The practical work done by the pupils includes some surveying, geometrical constructions, etc.

Teachers are free to choose the textbooks they wish to use.

Teachers

Secondary teachers of mathematics are required to hold the *agrégation* in mathematics, a first degree in mathematics plus some teacher training, or the diploma of the higher school of education.

Some difficulty is met with in recruiting secondary teachers of mathematics, partly on account of the war conditions which lasted until 1954, partly on account of the fact that secondary teachers are inadequately paid. To remedy the position it is proposed to establish scholarships, and to increase teachers' salaries.

Teachers secure further training through further training courses, reading the monthly journals concerning secondary education, and the advice of their inspectors.

YUGOSLAVIA

From the reply sent by the National Commission of the Federal People's Republic of Yugoslavia for Unesco

Introduction

Yugoslavia is a federative State of six republics. Each of these republics is entitled to draw up its own school curricula and syllabuses, and there are therefore some differences among these. The following report gives what aspects of the teaching of mathematics are common to all the republics, and indicates in what respects they differ.

Primary education lasts eight years, and caters for children up to the age of fourteen. General secondary education is provided in schools catering for pupils from 15 to 18 years of age, except for ten so-called " classical grammar " schools which are attended by pupils from 11 to 18 years.

Place of Mathematics in Curriculum

Mathematics is taught in all years of the general secondary and " classical grammar " schools.

The time-allocation for mathematics in such schools differs according to the type of school and the republic in which the schools are situated. A fairly typical example is that of the People's Republic of Slovenia, where from three to five hours a week (9.4% to 15.6% of the total timetable) is devoted to mathematics, or the People's Republic of Macedonia, where three or four hours (9.4% to 13.3%) is devoted to mathematics.

The subject is designated arithmetic and geometry in the 1st and 2nd year of the " classical grammar " school (corresponding to grades V and VI of the upper primary school), algebra and geometry in the 3rd and the 4th year (corresponding to grades VII and VIII of the upper primary school), algebra and geometry in the 5th and 6th year (grades I and II of the general secondary schools), algebra and trigonometry in the 7th year (grade III general secondary school) and algebra and analytical geometry in the 8th year (grade IV general secondary school).

Mathematics is a required subject in all years. No special importance is attached to mathematics as compared to other subjects, but, after the mother tongue, it has the highest time-allocation. In the secondary leaving examination mathematics is a compulsory subject. The marks for mathematics have the same value as the marks for other subjects.

Aims of Mathematics Teaching

The main aims of mathematics teaching are to impart the mathematical knowledge needed for general education and further study, to develop the pupils' reasoning power and understanding of functional relations, to

give clear ideas of space, and to foster habits of orderliness, accuracy, and perseverance. Other aims include relating mathematics to other subjects and to practical life, in regard both to simple everyday operations and to complicated calculations. Mathematical terminology helps to develop the pupils' ability to use language accurately, and mathematical symbols develop their capacity to think in general terms, and their power of abstraction. The notion of functional relations widens the pupils' mental horizon. Mathematics, in short, is regarded as an essential means both for mental training and for concrete technical achievement, as also for further study at higher level.

Syllabuses

1st and 2nd years of " classical grammar " schools (corresponding to grades V and VI of the primary school). Numerical calculations, and fundamental concepts of geometry. — *3rd and 4th years* (corresponding to grades VII and VIII of the primary school). Algebra : transformations, equations, areas of two-dimensional and three-dimensional figures, the theorem of Pythagoras. — *5th year.* Algebra : general fractions, linear equations, linear functions. Plane geometry (cont.). — *6th year.* Algebra : quadratic equations, powers, roots, logarithms. Solid geometry. — *7th year.* Algebra : quadratic equations. Trigonometry : transformations and equations. — *8th year.* Algebra : arithmetical and geometrical progressions, revision of functions and finite processes. Plane coordinate geometry. History of mathematics.

In the People's Republics of Slovenia, Croatia, and Serbia, concepts of derivates are introduced in the 8th year. The principles of axioms are introduced in the 5th year (grade I of general secondary schools).

Methods of Teaching Mathematics

Official instructions concerning methods of teaching mathematics at secondary level are issued only by the People's Republics of Macedonia, Montenegro, and Bosnia-Hercegovina. According to these instructions, mathematics should be taught during the lower cycle by means of inductive methods in which the principle of observation and activity of the pupils is applied. Deductive methods and strictly logical proofs are gradually introduced in the upper cycle. Similar methods to these are in use in the republics in which no official instructions are issued.

In regard to the transfer stage from primary to secondary education, instructions have been issued with a view to facilitating the change from the class teacher to the specialist teacher. Correlation of mathematics with other associated subjects is effected by so constructing the syllabuses that the mathematical knowledge required for understanding those subjects is suitably provided in advance, and by giving the pupils problems to solve which are taken from life and those subjects.

The teaching materials used for mathematics lessons include a variety of geometrical models. Use is also made of films.

Textbooks are approved by special commissions attached to the Councils of Education and Culture of the various republics. A textbook approved in one republic may be used in any other republic.

Teachers

Mathematics teachers for the upper cycle of secondary education are required to have completed eight semesters of university study, and to have graduated from the faculty of natural science and mathematics. During two semesters of the course, they take general pedagogy and methods, visit secondary schools, and do practice teaching.

Mathematics teachers for the lower cycle of secondary education (and the upper primary school) are required to have completed a two-year or three-year course in teaching training, in which the study of pedagogy and mathematics is compulsory, and which concludes with an examination.

There is a shortage of secondary mathematics teachers. This is due to the rapid increase in the number of schools. To overcome the shortage new teacher-training schools at higher level and new faculties have been opened, and a number of districts and towns offer scholarships to candidates for mathematics teaching.

Secondary mathematics teachers have several possibilities for further training. The associations of mathematics and physics teachers are very active, and organize consultations, courses, and seminars, and publish periodicals and brochures on the teaching of mathematics. Within the schools there are working groups in which mathematics teachers have opportunities of coordinating their work and exchanging experiences.

Miscellaneous

The general reform of education is under consideration, and will affect the mathematics syllabuses. These will be in part curtailed, and in part expanded with material related to the needs of modern life. The syllabuses will include, for instance, probability calculus, statistics and vector calculus.

The revision of mathematics teaching is also being considered by the mathematics teachers' associations, the federal institute for educational research, the education associations of the various republics, etc.

The periodicals published by the associations of mathematicians and physicists include the following: "The Teaching of Mathematics and Physics" (Yugoslavia), "Journal of the Association of Mathematicians and Physicists of the People's Republic of Serbia", "Review for Mathematics and Physics" (Slovenia), and "Journal of Mathematics, Physics, and Astronomy" (Croatia). The association of mathematicians and physicists, moreover, of Croatia, publishes a periodical for secondary pupils, called "The Gazette for Mathematics and Physics".

Publications of the International Bureau of Education *(cont.)*

2. Annuaire international de l'Education et de l'Enseignement 1938 . . . . . Out of print
3. Le Bureau international d'Education en 1937-1938 Fr. 1.50
4. VII[e] Conférence internationale de l'Instruction publique. Procès-verbaux et résolutions . . . . . Fr. 4.50
5. Annuaire international de l'Education et de l'Enseignement 1939 . . . . . Out of print
6. La rétribution du personnel enseignant secondaire Fr. 9.—
7. L'enseignement de la géographie dans les écoles secondaires . . . . . Fr. 7.—
8. L'organisation de l'éducation préscolaire . . . . Fr. 7.—
9. Le Bureau international d'Education en 1938-1939 Fr. 1.50
10. VIII[e] Conférence internationale de l'Instruction publique. Procès-verbaux et résolutions . . . . . Fr. 4.50
11. Le Bureau international d'Education en 1939-1940 Fr. 1.50
12. L'organisation des bibliothèques scolaires. . . . Fr. 7.—
13. L'éducation physique dans l'enseignement primaire Out of print
14. Le Bureau international d'Education en 1940-1941 Fr. 1.50
15. L'enseignement ménager dans les écoles primaires et secondaires . . . . . Fr. 9.—
16. The International Bureau of Education in 1940-1941 . . . . . Out of print
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